# A MULTI-SCALE MORPHOLOGICAL APPROACH FOR DELIMITING URBAN AREAS

*WORKINGPAPER PRESENTED AT THE CONFERENCE "COMPUTERS IN URBAN PLANNING AND URBAN MANAGEMENT", LONDON, JUNE 29 – JULY 1ST*

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**Abstract:**
In urban planning, the definition of limits of towns or of town sections is required in particular in the context of zoning. Such definition is purely conventional and does not refer to the morphological reality of urban areas. Even if it is possible to identify limits of zones by means of *functional criteria*, until now no appropriate method exists to define limits on the base of *morphological criteria*. This lack becomes particularly obvious when tackling with periurban areas. Thus, it is here proposed to analyse urban limits through a multi-scale approach, which makes it possible to study urban patterns by avoiding the introduction of any presupposition on the urban spatial structure.

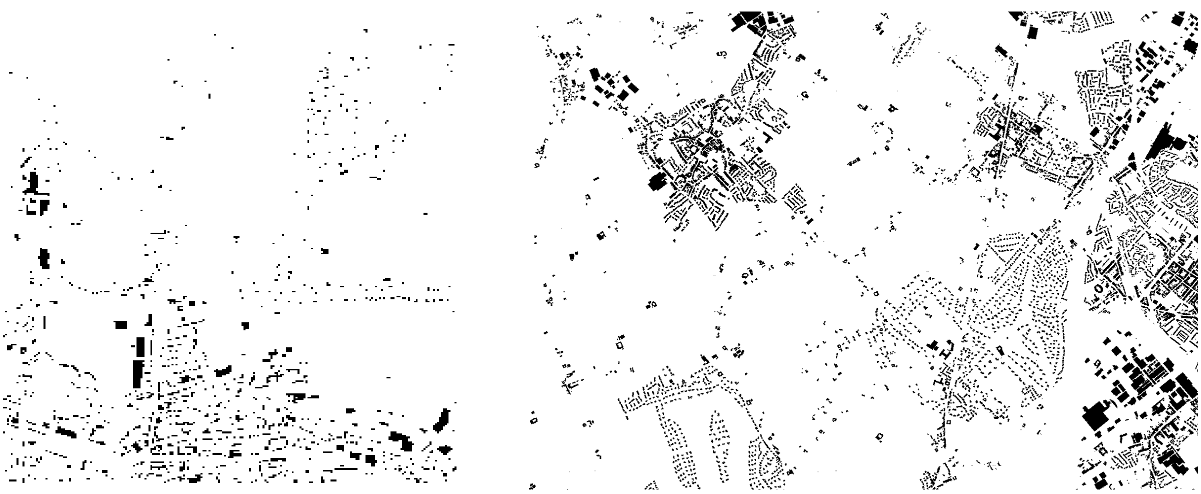
Different notions are introduced and discussed. The terms of "border", "limit" (or "boundary") and "envelope" are precisely defined and it is shown that if the urban border can in any case be identified, the external envelope of an urban area does not always exist. These reflections are illustrated by geometrical models on fractal nature.

In a first section of this paper, a fractal reference model of urban patterns and borders is introduced. On this basis, the second section of the paper is dedicated to the presentation of a coherent concept the envelope of an urban area, which leads to develop a methodology for extracting the urban envelope from cartographic representations of built-up areas. The third part of the presented research focuses on how fractal measuring methods may be fruitful for linking the notions previously defined to reality. On the one hand, the notion of urban border refers to the contours of buildings, which can be extracted from cartographic representations and which can afterward be analysed. On the other hand, the envelope of a zone is obtained by creating clusters of buildings lying within a certain range of distances. By considering stepwise larger and larger distances, bigger clusters emerge. At each step of the analysis, the envelope of the created clusters can be extracted and their morphology can be characterised by fractal measures.

## Introduction

In urban planning, the definition of limits of towns or town sections is required in particular in the context of zoning. Indeed, delimiting zones in an urban area or defining the boundaries of an urban area is an important challenge for applying planning policy, for collecting taxes, etc. However, no consensus exists about how to define the limit of an urban area. Each country recurs to its own criteria often combining different indicators, which is the source of ambiguities when comparing urban areas at an international level (Le Gléau *et al.*, 1997). Moreover, those definitions are purely conventional and do not refer to the morphological reality of urban areas.

It seems possible to find a way for defining limits of *densely built-up areas,* since the distances separating neighbouring buildings lie within a rather restricted range. To give an example, French authorities define “urban units” (*unités urbaines*) as the aggregation of buildings which are separated by less than 200 meters from one another and which belongs to either a “commune” (*i.e.* the smallest French territorial unit) or a set of “communes” with more than 2000 inhabitants. In Great-Britain, a more complex definition is used, which requires the existence of a transportation network and refers to buildings which are no more than 50 meters apart (Longley & Batty, 1991). However, such criteria become questionable when considering urban fringes. Irregular patterns occur, where recent detached housing estates and traditional rural settlement patterns are mixed. Figure 1a shows an example where urbanisation is very diffuse on the fringe of the town. This example shows that the distance between neighbouring buildings may differ considerably. In the example of figure 1b, a similar phenomenon can be observed at the level of a metropolitan area.



**(a)** **(b)**

**Figure 1: (a)** ***The northern urban fringe of the city of Besançon (East of France): diffuse strips of houses follow the road network, where ancient farm houses and new houses are mixed.*** **(b)** ***The western part of the urban area of Lille (North of France): different kinds of urban pattern are mixed.***

As suggested by the two examples presented in the figure 1, the delimitation of urban areas leads to carry out some reflections about the distinction between urban and rural. Actually, it can be considered that because urban areas include periurban areas, the delimitation of urban areas consists in the identification of the spatial extend of the periurbanisation[1]. But, it is as much difficult to find a univocal definition of an urban area as to find a univocal definition of a rural area. Two main reasons explain such a difficulty: firstly, there exists a wide range for the distances separating neighbouring buildings in sprawling urban patterns; secondly, the morphological characteristics of periurban zones may vary a lot from a much diffused urbanisation ("moth-eaten" rural areas) to reduced space consumption (grouped houses).

If it is possible to identify periurban zones on the base on *functional criteria* with respect to the typologies of intra-urban zones defined by different countries of Europe, until now no appropriate method exists to define limits on the base of *morphological criteria* (Caruso *et al.*, 2001; Caruso *et al.*, 2003). Morphological criteria are although particularly reliable since they refer solely to the spatial configuration of the considered pattern. On the contrary, functional criteria, because they are based on changing spatial units do not allow analysis across time. Moreover, in the perspective of an international comparison, it is interesting to define an urban limit on the basis of few criteria, which is possible using morphological criteria. Also in a modelling perspective, a morphological approach of the urban limits seems interesting, as pointed out by Batty and Longley (1986) *« In designing the models, it was thought important to keep the variables in the models as simple as possible and, at the same time, easily measurable ».*

Due to the difficulty to find morphological criteria for delimiting urban areas, it seems not surprising that for a long time little research tackled with the particular morphological features of urban patterns issued from urban sprawl. Of course, the urban sprawl phenomenon it-self has been widely studied, as well synthesised in (Galster *et al.*, 2001). But, as explained in (Torrens & Alberti, 2000), "*without robust empirical metrics to inform the debate, however, much of this argument remains conceptual, even speculative*". Moreover, as it is the case for the paper of Torrens and Alberti, research about morphological aspects of urban sprawl mainly aim to characterise (even to evaluate) its attributes and not to delimit the spatial extent of periurban areas.

Even the vocabulary used is sometimes ambiguous: different terms are used like envelope, limit, border, boundaries, fringes... But there meaning is not always the same. It appears here necessary to clarify some notions we use further in the present paper.

- The envelope is for us a virtual limit of the urban area. The reflections presented in the paper concern mainly the possibility of finding morphological criteria to define the envelope of an urban area. The main feature we consider for the envelope is how tortuous it is.

[1] We can here notice that the notion of *periurbanisation* does not exist in the English speaking literature, where the notions of *suburbanisation* or *counterurbanisation* are most often used (Caruso et al., 2001). Nevertheless, we chose to use here the concept of *periurbanisation* taken from French literature. Periurban areas are located between city and countryside. They can be defined by 2 major characteristics: they are under urban influence and they present rural characteristics (existence of an agricultural and forest system). The crucial difference between suburban and periurban areas is the presence or not of such rural characteristics.

- On the contrary, we call “borders” all really existing contours of objects. In a medieval town, the outer wall would be a border but could also be the envelope, if no suburbs exist outside.
- Sometimes, we also speak about “urban fringes” as we already did previously. We designate by this term the outskirts lying in periphery of the urban areas, where the urbanisation process transforms progressively rural settlements into periurban patterns.
- We use the expressions “urban limit” or “urban boundary” for characterizing either a border or an envelope in a rather general sense.

According to our terminology, the morphological definitions of urban limits, which refer to arbitrarily fixed criteria as distances between neighbouring houses, should be interpreted as some attempts to define an envelope. Indeed, such limits mostly do not really exist; one exception would be that of a medieval town, surrounded by a wall, without suburbs; the wall is then a real border which may be used as reference for defining an envelope.

Beyond planners and politicians, cartographers tend also to “materialise” the envelope when drawing coarse-grained maps on small scales. Nevertheless, cartographic lines are no real existing objects. Figure 2a illustrates this assertion: we designate as border of the agglomeration the set of all the limits of the buildings, as it may be obtained using a GIS, whereas the thick borderline corresponds to the virtual delimitation as it appears on coarse-grained maps like that of figure 2b.

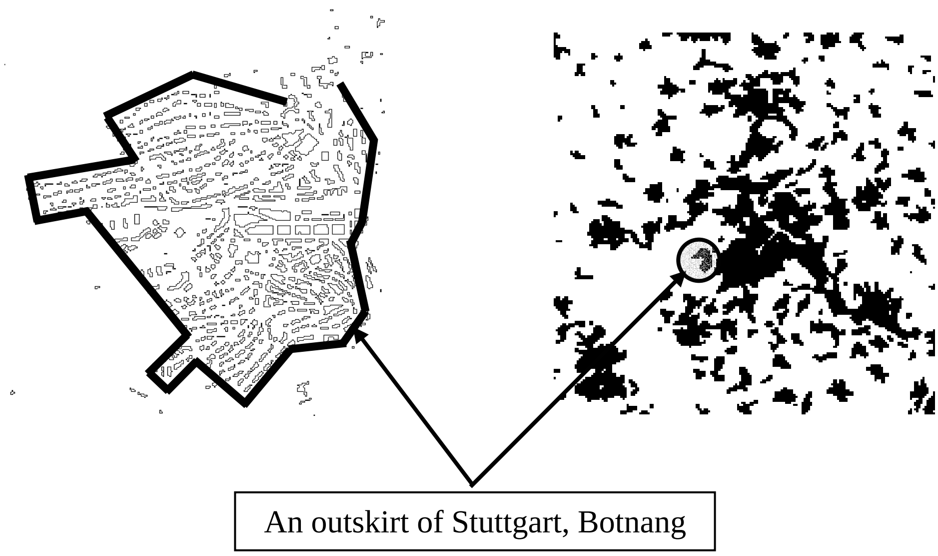



**Figure 2: (a)** ***An outskirt of Stuttgart, Botnang: the limits of the buildings have been extracted from a GIS data set based on cadastral maps. The envelope is a virtual line delimiting approximately the built-up area.*** **(b)** ***A simplified cartographic representation of the Stuttgart agglomeration. The borderline of the black spot referring to Botnang may be interpreted as an envelope resembling to the thick borderline of figure*** **(a).**

We could expect that such envelopes introduced by cartographers are more or less the result of neglecting details which refer to finer scales than the considered one. Eliminating the details would then be a purely morphological approach and could be of interest for finding a coherent way to define the envelope of an urban area. However, it is well-known that cartographers combine different methods to construct their simplified maps. Moreover they prefer representing symbolically important features of a given urban pattern to the detriment of a rigorous simplification of the original cartographic data[2]. Therefore, the creation of simplified maps is based on practical definitions, which does not provide a coherent representation of urban envelopes.

Hence, the present paper aims to expose preliminary reflections before proposing a morphological definition of urban areas. No precise definition is finally proposed, but the basis for such a definition is set out. On a substantive point of view, the goal is to develop a method, which can be used for constructing stepwise urban borderlines solely based on the spatial distribution of the buildings. The developed method refers directly to fractal geometry, which is by definition a multi-scale approach of forms. It allows us to take into account phenomena referring to different ranges of distances.

In a first section of this paper, a fractal reference model of urban patterns and borders is introduced. On this basis, the second section of the paper is dedicated to the presentation of a coherent concept the envelope of an urban area, which leads to develop a methodology for extracting the urban envelope from cartographic representations of built-up areas. In the third part of the paper, it is shown how such a method can be applied to real world patterns.

[2] E.g. they conserve important road axis by enlarging them artificially.

# 1. Bases for the reflection

As pointed out previously the main difficulty for finding a reliable procedure for defining an urban envelope is related to the fact that distances between neighbouring buildings vary within a large range, in particular in the fringes of agglomerations. This incited us to recur for this goal to a multi-scale approach based on fractal geometry in order to establish a link between the border of the town as defined previously and the construction of an envelope from a purely morphological point of view. For this aim we introduce fractal models, which serve to illustrate urban pattern morphology across scales

Only after this first step, we consider to what extend this envelope is tortuous or not. This question has already been considered in an early paper of P. Longley and M. Batty (1991) where the authors measured by means of fractal dimensions the shape of urban boundaries as they were defined by the Office of Population Censuses and Surveys. In this paper the authors called these urban boundaries "envelope".

## 1.1 The data used

We choose to distinguish three basic types of urban land-uses: buildings, transportation networks and free spaces like courtyards, squares etc. We consider transportation networks as non built-up space: built-up space consists solely of the buildings. Buildings are either isolated the ones from the others (*e.g.* detached houses), or they form clusters which are surrounded by the street network (*e.g.* terraced houses). Thus, on a morphological point of view, an urban area is composed by a multitude of clusters of different size.

## 1.2 The basic reference model: the Fournier dust

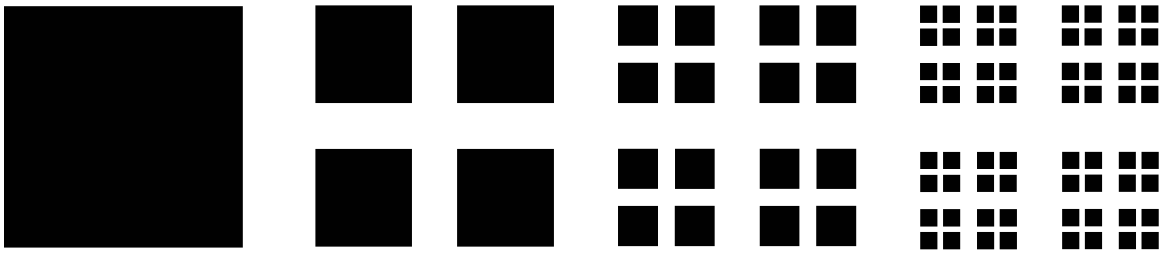
In order to develop a coherent fractal approach of the urban envelop, we recur here to a particular type of fractals showing properties, which seem adapted to describe the main morphological features of urban patterns relevant in this context. In many previous papers we used Sierpinski carpets as basic model for urban patterns (cf. *e.g.* Frankhauser (1994), Frankhauser (1997)). Here we will first recur to another type of model, the Fournier dust, which shows however many similarities with the Sierpinski carpets. Indeed in both cases the fractal is generated by transforming subsequently an initially given figure, the initiator. In a first step, a construction rule is defined which generates a patterns consisting of a certain number of smaller replicates of the initially given figure. In the further steps, this rule is applied to each of these replicates, which we call "elements" of the fractal. The construction rule, called generator, defines three properties of the fractal object:

- the factor $r$ by which the initial figure is reduced,
- the number $N$ of elements which are generated,
- the position of the elements which however do not affect the fractal dimension of the object[3]

Hence such fractals are based on an iterative construction principle. Figure 3 shows the first construction steps for a Fournier dust. The only difference between Fournier dusts and Sierpinski carpets is the position of the elements in the generator: in Fournier dusts all the

[3] The position of the elements may change in course of iteration, if the lacunas generated in the previous iteration steps are not affected.

elements are disconnected, whereas in Sierpinski carpets all elements are connected (at least by one point) and form thus one unique cluster.



the initiator first step : the generator second step

**Figure 3:** ***the iterative construction of a Fournier dust. We observe that the elements in the generator are disconnected.***

The example of figure 3 shows that Fournier dusts consist of a great number of isolated elements. Due to iteration the elements are not distributed uniformly on the given area; they form clusters, which are separated by empty lanes of different size: there exists a few number of large lanes and an increasing number of smaller and smaller ones. The relation between the number of lanes and their size follows a strong hierarchical law, which corresponds to a Pareto-Zipf distribution, well known in geography and economics (Tannier & Pumain, 2005).

**Figure 4:** ***Comparing two types of generators of Fournier dusts with urban patterns: In the upper example the urban pattern consists of terraced houses forming blocks of houses, surrounded by street. This main feature reappears in the fractal. In the other example the buildings are more detached what has inspired to propose another type of generator.***

In order to illustrate that Fournier dusts are rather well adapted to illustrate typical features of urban patterns we have in Figure 4 compared two other Fournier dusts in second iteration with real world urban patterns.

# 2. From border to envelope using fractal geometry

Let us now focus on the question whether it is possible to apply the concept of border and of envelope to such an object and how these two concepts may be linked.

## 2.1 The borders of a Fournier dust: what changes and what remains at each iteration step?

Considering one given step of iteration, it seems easy to identify the boundaries of a fractal object: they correspond to the contours of the elements, which are squares in our examples. But in course of iteration the borders change, as the elements change themselves: on the one hand the number of elements is multiplied by $N$ at each step and increases therefore according to a geometric series. Hence, for a given step $n$, the number of elements $N_n$ is:

$$N_n = N^n$$

On the other hand, at each step the size of the elements is reduced by the factor $r$ and thus the base length of the elements $l_n$ decreases according to a geometric series, too:

$$l_n = r^n \cdot l_0$$

Assuming that our elements are square-like, the boundary length of one element would be for a given iteration step $n$:

$$p_n = 4 \cdot l_n = 4 \cdot l_0 \cdot r^n$$

and thus would tend to *zero* for $n \to \infty$:

$$\lim_{n\to\infty} p_n = 0$$

This may be understood by the fact, that the iteration generates for $n \to \infty$ a set of isolated points, which are concentrated in a multitude of clusters. Thus, the boundary of a single element vanishes. Considering all the elements constituting the studied form, we can define the length of the cumulated boundaries $P_n$ as following:

$$P_n = N_n \cdot p_n = 4 \cdot N_n \cdot l_n = 4 \cdot l_0 \cdot N^n \cdot r^n = 4 \cdot l_0 \cdot (N \cdot r)^n$$

Thus three cases may be distinguished

- $N \cdot r > 1 \Rightarrow$ the cumulated perimeter $P_n$ diverges
- $N \cdot r = 1 \Rightarrow$ the cumulated perimeter $P_n$ is constant
- $N \cdot r < 1 \Rightarrow$ the cumulated perimeter $P_n$ converges to zero

Hence, even if the concept of border is delicate to handle for fractal forms like Fournier dusts, it is although possible to define a border for each iteration step.

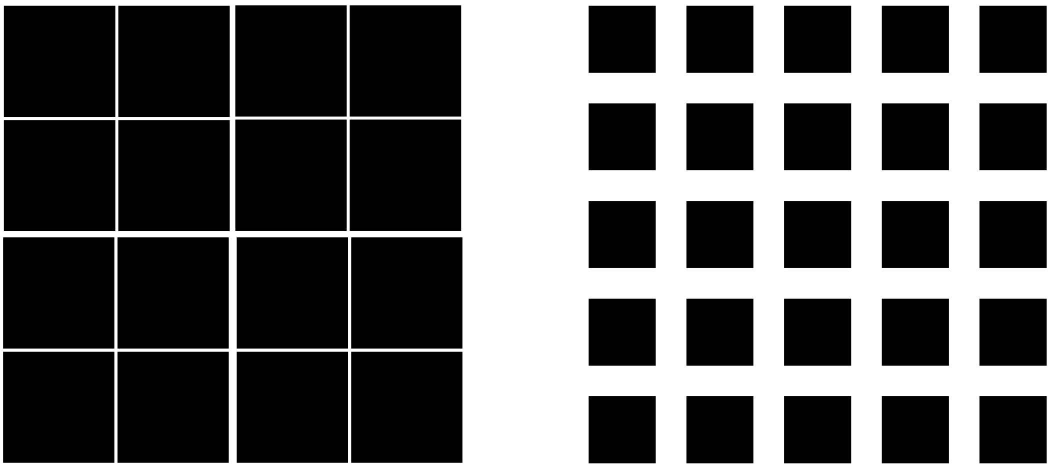
The notion of fractal dimension, presented in details in multiple papers (*e.g.* Mandelbrot 1982) allows however introducing a measure, which characterizes the borders by one unique value for all iteration steps. For constructed fractals like that of figure 3 the fractal dimension $D$ of the fractal object may directly be linked to both the parameters $N$ and $r$ by means of the relation

$$D = -\frac{\log N}{\log r} \quad \text{yields} \qquad N = r^{-D}$$

This fractal dimension is called *self-similarity dimension*. Fractal dimensions allow relating the number of elements $N_n$ to the size $l_n$ of the elements by the following relation:

$$N_n \sim l_n^{-D}$$

The fractal dimension $D$ characterizes the non-uniform distribution in a fractal of the elements across scales. Dimensions close to $D = 2$ correspond to fractals where mass is nearly distributed homogeneously. Figure 5a shows an example of such a Fournier dust in second iteration. Since the free lanes are very small, their widths vary rather slowly in course of iteration, and hence the system of free space will not be very hierarchical.



(a) (b)

**Figure 5: (a)** ***A Fournier dust in second iteration which resembles to that of figure 1. However in this example, the width of the lanes doesn't vary much from one step to the next one. Hence, the fractal dimension would be close to D*** **= 2. (b)** ***A completely uniform pattern, without any hierarchy: all the lanes have the same width and thus the fractal dimension is D*** **= 2.**

The value $D = 2$ corresponds to a completely homogeneous distribution, which does not mean, that we have a uniform black surface. Figure 5b shows an example, which is not a generator of a fractal, but where we have just put squares of the same size together, separated by lanes of equal width. Since the squares are distributed in a uniform way, the fractal dimension of this geometric object is $D = 2$.

The other limit case corresponds to an isolated point, which has of course the fractal dimension $D = 0$. Intermediate values correspond to situation where the distribution of the lane width is more or less hierarchical.

We have seen that the usual perimeter lengths $p_n$ and $P_n$ vary according to the considered iteration step. By recurring to fractal dimension, we may define a generalized perimeter length $\mathcal{P}$ which is the equivalent to $P_n$ but which remains constant all over the iteration steps:

$$\mathcal{P} = N_n \cdot ( p_n )^D = (4 \cdot l_0 )^D \cdot N^n \cdot r^{Dn} = (4 \cdot l_0 )^D \cdot (N \cdot r^D)^n = (4 \cdot l_0 )^D$$

… where we have used the relation $N = r^{-D}$.

In fact, $\mathcal{P}$ is the perimeter length of the initiator of the considered fractal form (*i.e.* the initial square in the figure 3). Hence the Fournier dust may be completely characterized by both the constant parameters $\mathcal{P}$ and $D$. In the following we will get aware, that these parameters allow linking the notion of border to that of envelope to which we will come back now.

## 2.2 Generating the envelope

Since all the elements of a Fournier dust are disconnected, defining an envelope seems difficult – at least from the point of view of Euclidean geometry. However, we will see that the iterative nature of fractal geometry will help us to introduce this notion and to visualize the envelope according to a clearly defined procedure. For this aim we consider first the Fournier dust of figure 3 for which we have represented the first and second step in figure 6a and 6b. The figure 6a corresponds to the generator. The width of the free lanes $\lambda_1$ is in this example the same as the base length $l_1$ of the elements (Figure 6b). We put now at a given iteration step *i* on each element another square, centred on the element (figure 6c and 6d). In measure theory, such a procedure is called “covering” a structure by test elements of size $\varepsilon_\iota$ The test elements of figure 6c are squares whose base length is just $\varepsilon_I \equiv l_1' = 2\ l_1$. Then each of the new squares touches its neighbours and we have now one unique cluster. Hence it is possible to define an envelope for this new object: it is just the exterior perimeter of the four squares, which form a cluster. The total perimeter length of this cluster is $P_1' = 16\ l_1'$.

The figure 6d shows the same method applied to the next step. We observe that now 4 disconnected clusters are obtained if we choose for the covering squares a size, which eliminate just the newly generated smallest lanes between the squares. We may again cover these clusters by one unique larger square which touch just one another (dashed line) and which is just identical with the square with fat borderline of figure 4c. Thus, we come progressively back to the envelope we found for the first step of covering.

We should now emphasize that the envelope we constructed step by step, may be identified as iteration steps of another Fournier dust. The progressive construction of this Fournier dust is illustrated in figure 7. The only difference between the two generators is the position of the elements. In the first case we considered, the generator consists of four squares of base length $l_1 = r \cdot l_0$ , placed in the *corners* of the initial square (cf. figure 6a), whereas in the second case the four generated squares of length $l_1' = r \cdot l_0'$ are placed in the *centre* of the initial square and form a square-like cluster (cf. figure 7). The length of the initial square $l_0'$ is again twice the length $l_0$: $l_0' = 2\ l_0$. This large square is indicated on figure 6c and 7. Using the relation between $l_1$ and $l_1'$ we obtain;

$$l_1' = 2\ l_1 = 2\ r \cdot l_0 = r \cdot l_0'$$

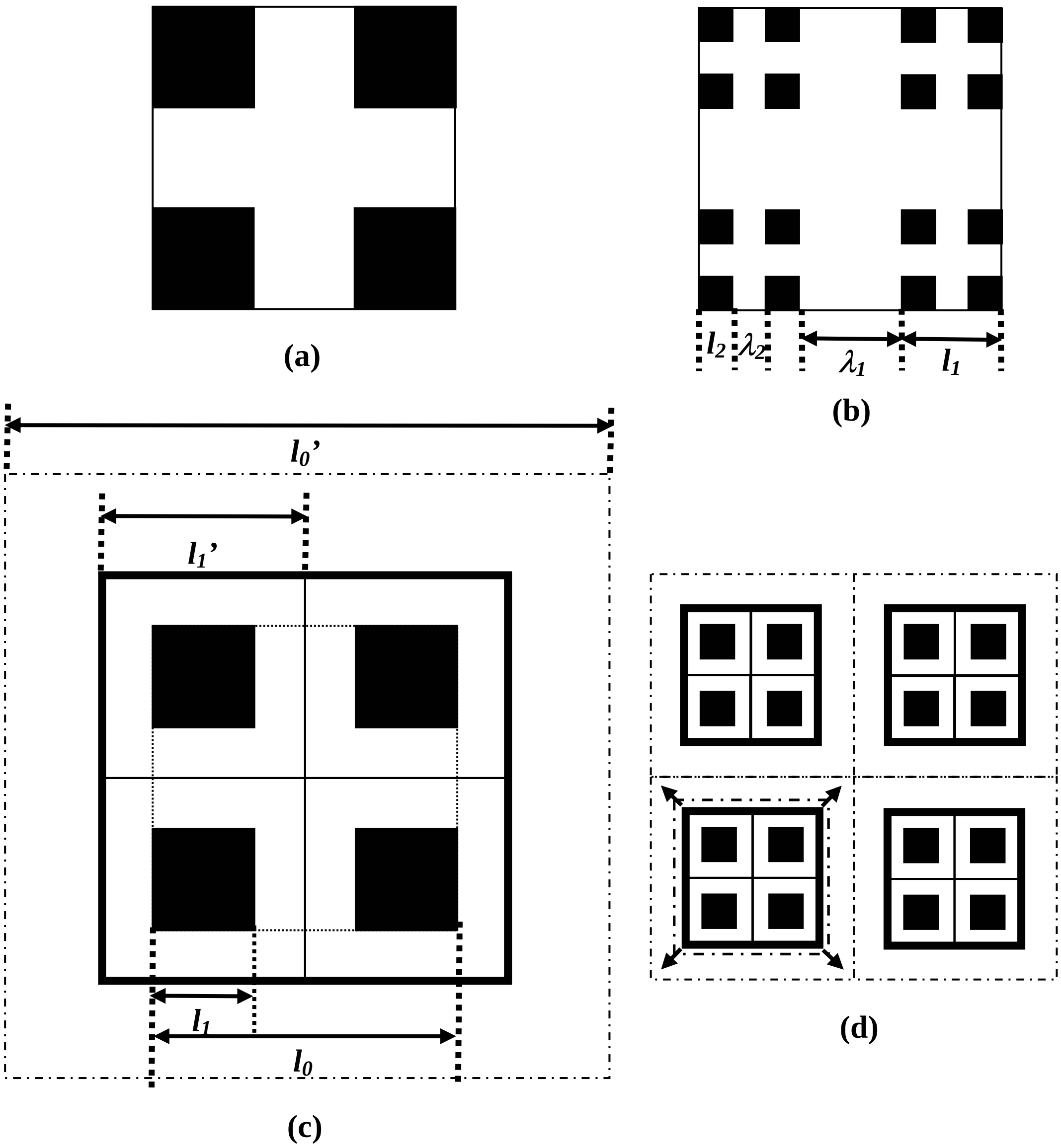



**Figure 6: (a) and (b):** ***The two first steps for generating the Fournier dust of figure 1. The widths of the lanes generated are indicated.*** **(c) and (d):** ***The covering of the Fournier dust of figure 1 by another Fournier dust. This second dust covers at each step just the lanes generated at the same iteration step for the first dust. The respective lengths of the elements are indicated, where the prime refers to lengths of the second dust.***

Thus we find the same scaling relations for $l_1'$ and $l_l$ . On the other hand we have already seen that the number of elements $N$ is identical, too, for both the Fournier dusts. ***Hence the both Fournier dusts have the same fractal dimension, which leads to conclude that generate an***

***envelope using the covering method appears as being a consistent method for delimiting urban areas.***

The presented approach generates two different kinds of information:

- the notion of fractal dimension allows to describe the progressive passage from border to the envelope. The fractal dimension $D$ characterizes how the cumulated perimeter $P_n$ changes in course of iteration.
- the extraction of the envelope allows defining a limit of the urban cluster entirely based on a morphological approach.

The form of the generated envelope is a very simplified approximation of a structure, which loses finally its multi-scale properties when coming back to the initiator.

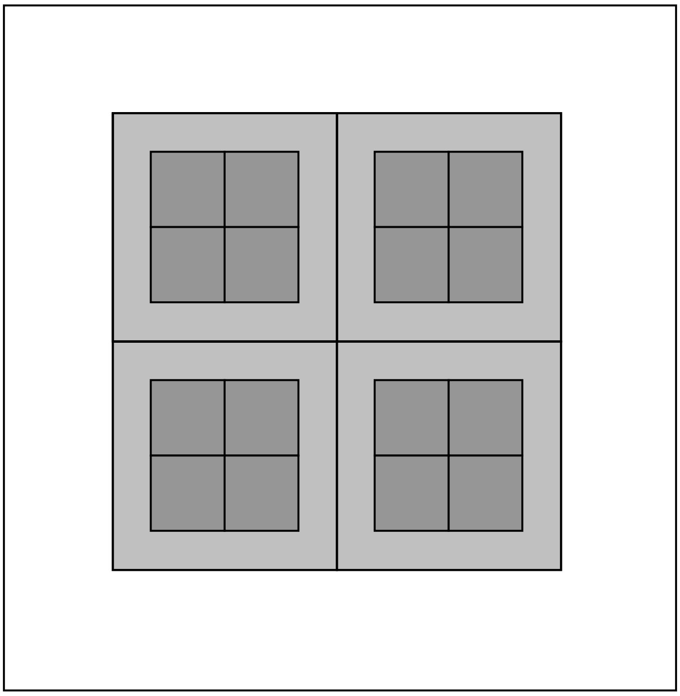


**Figure 7:** ***the iterative construction of the second Fournier dust (cf. text), which covers at each step the dust of figure 4a, b in that way, that the smallest lanes disappear (cf. text and figure 4).***

Real urban patterns are of course more complex structures than the discussed Fournier dusts:

- Even if we assume a rather hierarchical organisation for the streets network of a city centre, we may not expect that this spatial system follows such a strong scaling behaviour as that of the lanes in the presented Fournier dusts.
- In suburban zones, free space separating individual houses do not follow a strong hierarchy.
- In the presented Fournier dusts, all the elements belonging to a given iteration step have strictly the same size, which is not true for buildings in a town.

Hence, for extracting the urban envelope it does not seem possible to apply the same covering method as previously described. This incited us to recur to an alternative procedure, the dilation method, for generating the envelope: when starting from a given iteration step we dilate stepwise the square-like elements until these dilated squares just touch their neighbouring dilated elements[4]. When applying this method to the second iteration step of the Fournier dust of figure 6 we obtain the situation represented in figure 6d: at the bottom at left we have shown how such a dilation works; finally the four dilated squares will form bigger squares, which are identical with that ones obtained by covering.

By going on with dilation, we obtain again the unique big square, when the four remaining squares join each other. Thus, we come back to the envelope we found for the generator using the covering procedure. Finally we introduced covering just in order to obtain a coherent approach for generating the envelope and for showing the possibility to describe the changes of the boundaries across scales by means of fractal dimension.

Let us give some details on the dilation procedure. We should be aware that after *i* dilation steps the perimeter length $p^{(k)}_i$ of a given cluster *k* (a square in the example of figure 4) has increased with respect to its initial length $p^{(k)}_0$ . For a simple situation like that of the squares of figure 4, we may easily verify the relation

$$p^{(k)}_i = p^{(k)}_0 + 8 \cdot i$$

This hold too for the cumulated length $P_i$ , which increases in a linear way with the dilation step *i*:

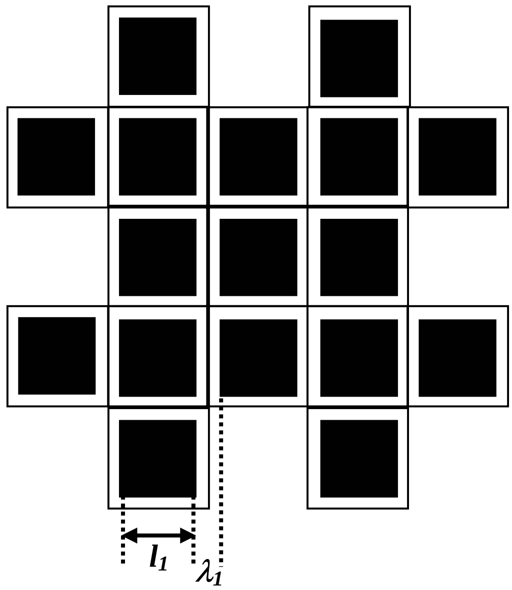
$$P_i = \sum_{k=1}^{K} p^{(k)}{}_i = \sum_{k=1}^{K} \left( p^{(k)}{}_0 + 8 \cdot i \right) = \sum_{k=1}^{K} p^{(k)}{}_0 + 8K \cdot i$$

*K* is the total number of clusters. When reaching the step where neighbouring clusters touch, the lanes separating these clusters disappear and thus simultaneously the borderlines. Then and the cumulated length $P_i$ will decrease abruptly. When considering situations like that of figures 4d and 4c, we get aware that the emergence of big clusters consisting of 4 smaller ones reduces the total perimeter length by the amount $\nu\, p^{(k)}_i$ where $\nu$ is the number of square-like clusters which have been generated. We may conclude that such sudden decreases of $P_i$ correspond to relevant steps for the emergence of the envelope.

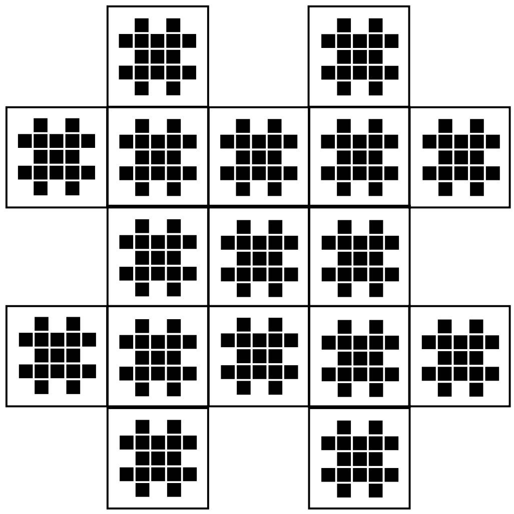
From a morphological point of view, the considered example is of course very simple. Thus we will tackle with a more complex Fournier dust, represented in figure 6. The initial figure is one more time a square. The generator consists of $N = 17$ squares and the reduction factor *r* is slightly inferior to 1/5 (figure 6a). This allows constructing a generator where lanes of width $\lambda_1$ separate the elements in the central part. We obtain for the lane width

$$\lambda_1 = ¼\ ( l_0 - 5 \cdot l_1 ) = ¼\ ( 1 - 5 \cdot r )\ l_0$$

[4] The dilation method was introduced by Minkowski who used it for exploring then scaling behaviour. More recently the so-called "mathematic morphology" recurs to dilation for identifying changes in spatial organisation (Serra J. 1988, for a geographical application cf. *e.g.* C. Voiron-Canicio 1997). Let us emphasize that in our paper dilation is just introduced for replacing covering and for extracting boundaries.

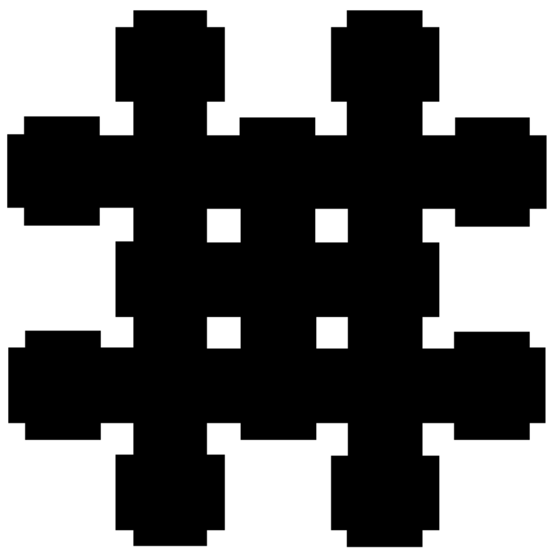



(a)



(b)

**Figure 8:** ***the generator* (a) *and the next iteration step* (b) *of a more complex Fournier dust. The covering of the generator is indicated in both the figures. Figure* (c) *shows the structure obtained by dilating such as the largest lacunas – those of the generator – just disappear. We see that the figure is not the same as that of covering***



(c)

In figure 8a the elements are again covered by squares, which just allow eliminating the small lanes of width $\lambda_1$. The length of base of the surrounding squares is then $l_1 + \lambda_1$ and we obtain one unique cluster, the boundary of which may again be considered as the envelope. However, we see that in this example the envelope is not identical with the initial figure, which is a square: ***in this case, the minimal covering does not generate the initial figure.*** This can be understood by observing the distribution of free spaces in the generator. In our first example (*cf.* figure 6), the only free spaces were the lanes separating the four squares. These lanes had all the same width $\lambda_1$. In the present example (*cf.* figure 8), this is no longer the case: by creating large free squares in the periphery of the generator, two scales of lacunas have been introduced. The proposed covering eliminates the smallest lacunas.

The presence of lacunas of different size influences also the dilation procedure. In the example presented on the figure 6, the dilation generated squares at each iteration step. This is no longer the case for the Fournier dust of figure 8. We have progressively dilated the second iteration step of the Fournier dust (figure 8b) just until neighbouring clusters join each other and one unique cluster appears (figure 8c). This iteration step corresponds to the situation obtained by covering in figure 6b, but the results are quite different: due to the fact that free spaces separating neighbouring elements are not equal in the generator, the smaller lanes in the centre of the generator are filled up after few dilation steps, whereas the larger lacunas disappear only after further iteration steps.

Finally, the stepwise procedure of dilation makes appear morphological particularities, peculiar to the scale under consideration: indeed, for step *i*, all details having a size $\varepsilon < 2\,i + 1$, have disappeared. Thus, the obtained envelope is an approximation of the considered pattern at this scale.

Let us now focus on the link between the two procedures we discussed, covering and dilation, and the notion of fractal dimension. We saw that in the first example, that of figure 6, we could interpret the covering procedure as the iterative construction of another Fournier dust, based on the same parameters *N* and *r* than the original one. Hence the fractal dimension of the obtained envelope was the same as that of the original Fournier dust**.** This logic of covering corresponds to the method proposed by Hausdorff and Besikovich for determining the fractal dimension of a structure (Mandelbrot 1982). For fractals constructed by means of an iteration procedure, like the discussed Fournier dusts usually the Hausdorff-Besikovich dimension is identical to the self-similarity dimension we introduced previously (see page 9), but this may not be the case for more complex structures (Mandelbrot, 1982).

Contrarily to covering, dilation does not provide a way for constructing an associated fractal having the same self-similarity dimension. Nevertheless, dilation is directly linked to the notion of fractal dimension too. Let us remind that Cantor, Minkowski and Bouligand recurred to dilation for determining fractal dimension (Mandelbrot, 1982). For this aim, they referred to the fact that an isolated point would be enlarged by dilation to a square of surface $\varepsilon^2 = (2\,i + 1)^2$ at step *i*. They determined for each dilation step *i* the filled up surface *S(i)* and divided that surface by $\varepsilon^2$. Thus they obtained just the number of squares $N(\varepsilon)$ of size $\varepsilon$ required to cover all points of the structure at this scale.

$$N(\varepsilon) = \frac{S(\varepsilon)}{\varepsilon^2} = \frac{S(\varepsilon)}{(2\varepsilon + 1)^2}$$

A fractal relation links the number of squares $N(\varepsilon)$ to the size of the reference square $\varepsilon$ :

$$N(\varepsilon) \sim \varepsilon^{D_{dil}}$$

## 2.3 From Fournier dusts to Sierpinski carpets

According to the previous discussion, we dilated stepwise the built-up surface of urban patterns (Figure 9). We get aware that, in many cases, the aspect of the patterns changed fundamentally after few dilation steps: as soon as the small streets and courtyards disappear, a reduced number of rather important clusters emerge, which have however irregular borders. Within these borders, we observe however empty lacunas of different size. This incites us to modify our fractal reference model in order to complete our insight into urban envelope. We come back to one model we used in a couple of previous papers as references for characterizing urban patterns, Sierpinski carpets (*cf. e.g.* Frankhauser 1994). The figure 9a reminds the aspect of the Sierpinski carpet.

As pointed out before Sierpinski carpets and Fournier dusts are constructed according to the same principle. However with respect to the question of the envelope, there exists a fundamental difference between the two types of fractals: since a Sierpinski carpet consists of a unique cluster at all scales, border and the envelope cannot be distinguished, contrarily to Fournier dusts.

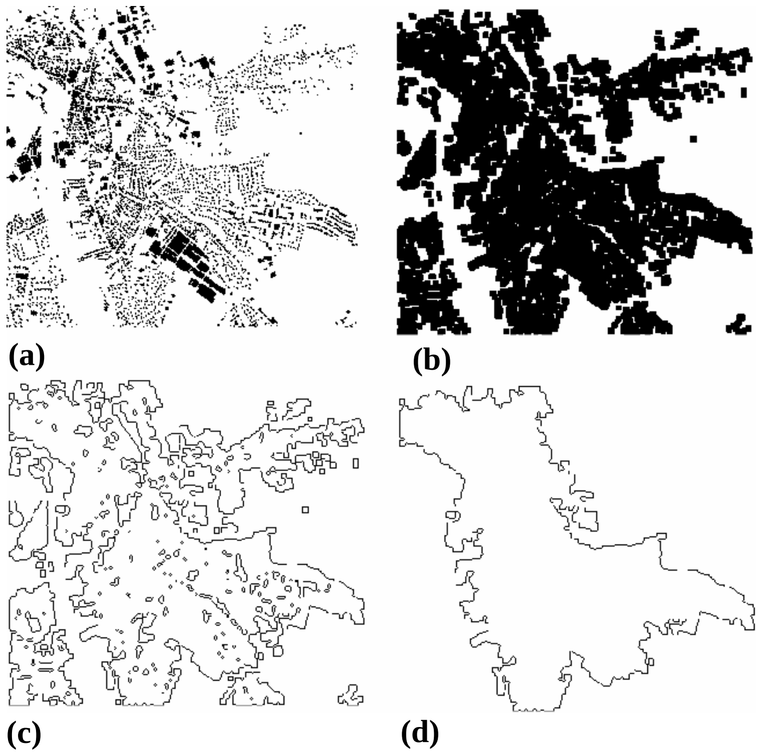


**Figure 9:** ***Extracting the outlines of a town by dilation of the built-up area*** **(a).** ***Figure*** **(c)** ***shows all the outlines, figure*** **(d)** ***shows the outline of the main cluster.***

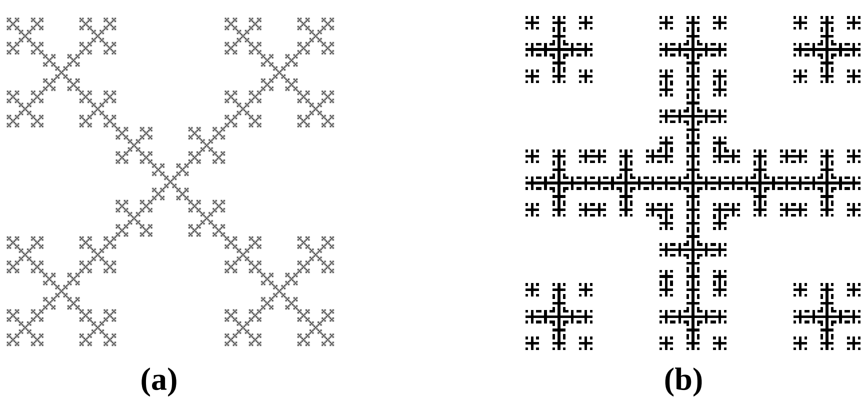


**Figure 10:** ***the Sierpinski carpet*** **(a)** ***consists of one unique cluster whereas the hydrid Sierpinski carpet consists of a series of clusters*** **(b)**

Figure 10 shows that both the models, Fournier dusts and Sierpinski carpets, may be combined. Such structures remind more the morphological aspect of urban patterns. We call this type of fractal form *hybrid Sierpinski carpets*.

Such hybrid structures show a particular type of hierarchy which exist neither in Sierpinski carpets nor in Fournier dusts: the generator consists of one important cluster in the centre and four isolated elements in the corners. In course iteration an increasing number of smaller and smaller clusters are generated in the vicinity of clusters generated at previous steps.

Let us now focus on another feature of the hybrid Sierpinski carpet which is this time linked to the central cluster of the generator which consists of $N^{(Sc)} = 9$ connected elements and forms thus itself the generator of a Sierpinski carpet. These elements are placed in a cross-like way and due to this arrangement the boundary of each cluster becomes more and more tortuous in course of iteration. Figure 11 shows the lengthening of the boundary for a part of this Sierpinski carpet.

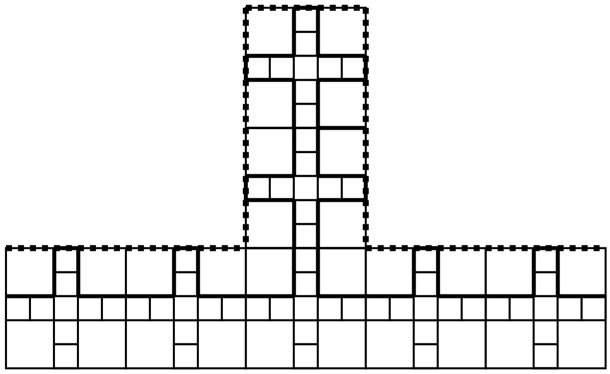


**Figure 11:** ***The upper part of the central cluster of figure 10. The dashed borderline corresponds to that of the generator and the fat line to that of the second iteration step.***

As for all Sierpinski carpets this subsequent lengthening of the boundary is characterized by the self-similarity dimension[5], which may defined in the usual way:

$$D^{(sc)} = -\frac{\log N^{(sc)}}{\log r}$$

Beyond this dimension it is possible to define a self-similarity dimension which refers to the whole hybrid Sierpinski carpet by referring to the total number of elements generated $N^{(hSc)}$. E.g. in the example of figure 8b the total number of elements of the generator is $N^{(hSc)} = 13$ and the reduction factor is $r = 1/5$. At each iteration step, 13 replicates of the generator are created, which generate either new isolated clusters or belong to already existing ones.

[5] At each step the base length of the square-like elements is reduced by the factor $r = 1/5$, and the number of elements is multiplied by $N^{(Sc)} = 9$. Due to the specific arrangement of the elements the lengthening of the boundary may slightly be inferior to that obtained by multiplying by the factor $N^{(Sc)}$. However mathematically this may be neglected since for Sierpinski carpets boundary and surface tend to the same limit set (Frankhauser 1994) and this is the reason why their fractal dimension is the same.

According to the usual definition the self-similarity dimension is defined in the following way:

$$D^{(hsc)} = -\frac{\log N^{(hsc)}}{\log r}$$

Hence this dimension $D^{(hSc)}$ globalizes information about the two different morphological phenomena enounced, the subsequent fragmentation of the fractal due to the generation of an increasing number of smaller and smaller clusters, and the just discussed lengthening of the boundaries of these clusters. We may say that this dimension characterizes the complexity of the structure.

***Thus both the dimensions $D^{(hSc)}$ and $D^{(Sc)}$ give complementary information about the boundaries: the first one characterizes the global complexity, the second one the tortuous shape of the clusters' boundary.***

As for all constructed fractals it is possible to link for each iteration step *n* the numbers of elements to their size $l_n = r^n\, l_0$. In the present case we obtain two such laws, one for the total numbers of elements $N_n^{(hSc)}$, and another one for the number of elements $N_n^{(Sc)}$ belonging to the Sierpinski carpet[6]:

$$N_n^{(hSc)} \sim l_n^{\;-D(hSc)}$$

$$N_n^{(Sc)} \sim l_n^{\;-D(Sc)}$$

It is now possible to eliminate the length of the elements $l_n$ common to both the relations, and to obtain thus a relation linking the total numbers of elements to that one of the Sierpinski carpet:

$$N_n^{(hSc)} = \left(N_n^{(Sc)}\right)^{\frac{D^{(hSc)}}{D^{(SC)}}}$$

This relation links the complexity described by $D^{(hSc)}$ to the lengthening of the boundaries, described by $D^{(Sc)}$. Hence it expresses just the supplementary contribution to complexity which is not due to lengthening the boundary. However we know that this supplementary contribution to complexity is just due to the emergence of the clusters which causes an increasing fragmentation of the structure in course of iteration. Thus we may call ***index of fragmentation*** the fraction $\phi = D^{(hSc)} / D^{(Sc)}$.

## 2.4 Extracting the envelope of hybrid Sierpinski carpets

Like for complex Fournier dusts, as for that one of figure 8, it is difficult to extract the envelope of hybrid Sierpinski carpets by means of covering: on the one hand the elements are adjacent in the clusters, on the other hand the different clusters are separated one from the other by empty space. However, the dilation allows to join progressively the closest clusters and to generate envelopes on different scales. In this case presented on the figure 12, the final envelope is a square which corresponds to the position of the elements in the generator, where the isolated squares are put into the corners of the square-like initiator.

[6] Mathematically spoken the Sierpinski carpet is a fractal subset of the hybrid Sierpinski carpet.

The figure shows that in course of dilation more and more coarse-grained approximations of the structure occur and we observe the emergence of larger and larger clusters. However we rediscover the typical features of this type of fractal structure: there exist still clusters of different size until the last steps where only one solely clusters subsists. This reminds typical morphological properties of real world urban patterns as shown on figure 9c.

In the next chapter we focus on the way how we may concretely apply this concept to real world patterns and what are the measures of interest for describing the morphology of urban boundaries.

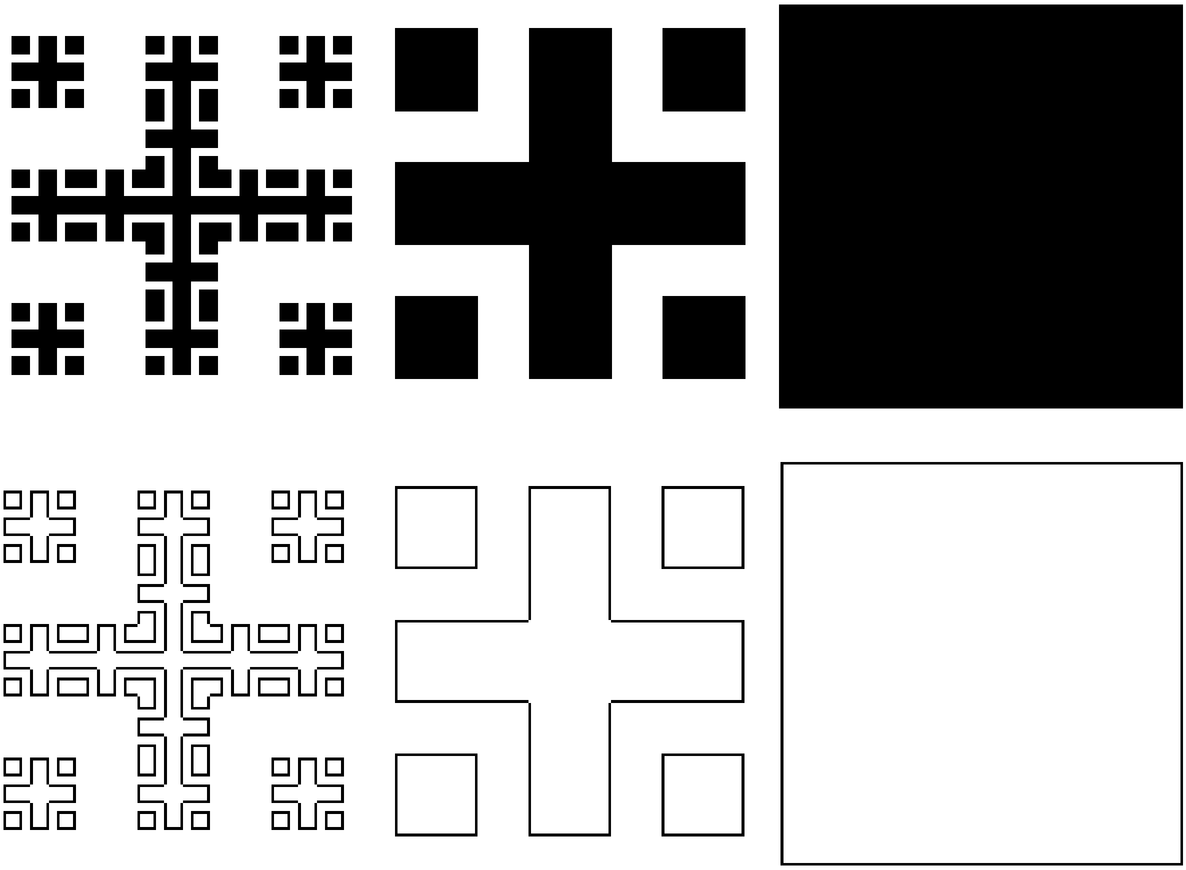


**Figure 12 :** ***Different steps of dilation of the hybrid Sierpinski carpet of figure 8 b (above) and the extracted boundaries which become finally the envelope (at the bottom, on the right). The central figure above corresponds to the generator of this fractal.***

# 3. Measuring the urban boundaries

## 3.1 The general methodology of analysis

The previous discussion made evident that we may expect different phenomena coming into play when trying to extract by means of dilation the urban envelope and to measure the morphology of the urban border and envelope. As pointed out, after little dilation steps, urban patterns tend to show features reminding hybrid Sierpinski carpets since they consist of a certain number of clusters of different size, which have more or less tortuous borders (cf. figure 7b). In course of dilation the clusters join progressively neighbouring ones and finally one unique cluster should subsist. In our concept the boundary of this final cluster corresponds to that we called the envelope of the urban pattern. In principle this envelope may be smooth as in the example of figure 12 but usually we may expect that this will not be the case for urban patterns since sprawling town are not contained by circular or square-like ramparts. We may even expect that like in the example of figure 2 the envelope has a tortuous form: there exist larger and smaller bays which enter in the build-up area. This reminds the example of figure 6, where the envelope of obtained for the dilation step which generated the unique cluster is still tortuous. Thus it seems of interest to measure the fractal behaviour of the envelope what reminds the early work done by P. Longley and M. Batty (1991).

But the example of the hybrid Sierpinski carpet made also evident that the envelope is not the only relevant morphological property for studying boundary phenomena in urban patterns. Different topics turned out to be interest:

1. The complexity of the pattern: even if the final envelope is smooth, the structure may be very complex on finer scales; it may consist of a great number of clusters of different size which have tortuous boundaries. The multi-scale aspect of this complexity is measured by the fractal dimension $D^{(hSc)}$.
2. The morphology of the clusters' boundaries: this could be described by the fractal dimension of the Sierpinski carpet $D^{(Sc)}$.
3. The fragmentation index $\phi$ which characterizes the fragmentation due to the presence of an increasing number of smaller and smaller clusters.

In constructed fractals, the fractal properties remain the same over the scales, thus the dimension values $D^{(hSc)}$ and $D^{(Sc)}$ are sufficient for characterizing the morphological properties. This may be different for real world patterns: perhaps there exist particular scales where the fractal behaviour of the extracted boundaries changes (e.g. the mentioned transition from a Fournier dust like pattern to a hybrid Sierpinski carpet). Thus it would eventually be of interest to analyse the fractal behaviour for different boundaries extracted in course of dilation.

This incites us to suggest the following procedure:

1. The given urban pattern is stepwise dilated and for each dilation step the boundaries are extracted, which form together the cumulated boundary.
2. For each step the cumulated perimeter length and the number of clusters is determined and stored;
3. The fractal dimensions of the cumulated boundary, as well as that of the different clusters' boundaries are determined for each step. In practice it would be possible to determine the boundary dimension for all clusters; we may restrict to the most important ones.
4. The dilation is stopped when a unique cluster occurs. In practice it turns out that we must be more flexible: going on with dilation until one very smooth cluster occurs may not be of interest and not traduce the reality. We will see that we may stop dilation when reaching a stable situation.

This procedure allows after having stopped the dilation to represent on a graphic the evolution of the border length and the number of clusters all over the dilation steps (*cf.* figures 12 and 13). If the fractal behaviour would not change over iteration, we should observe for the cumulated boundary length a power law according to the relation

$$P_n = N_n \cdot p_n \sim N_n \cdot l_n \sim l_n^{1-D}$$

In reality the mentioned passage from the Fournier dust to the hybrid Sierpinski carpet may disturb the regular decrease of the boundary length. If the structure follows fractal behaviour, we may expect a similar power law for the number of clusters. Indeed they should follow a hierarchical distribution law too, since they are generated by the iteration process.

## 3.2 Applying the methodology to urban patterns

For exploring urban borders end envelopes we should use detailed cartographic representations on the scale of buildings as it may be obtained from a GIS data base or eventually from certain topographic maps. The material source of the analyses is a raster image of an urban pattern. This image is composed of two types of pixels: black pixels for representing built-up areas and white pixels, which represent non built-up areas (free spaces).

For realizing all necessary operations like dilation or measuring fractal dimension we developed in the last years a specific software package for analysing the fractal behaviour of urban patterns, called *fractalyse*[7]. It offers different tools as dilating images, extracting boundaries as well as a couple of methods to measure the fractal dimension of an image. All generated images (dilated patterns, boundaries) can be stored as well as all information concerning the fractal analyses realized. For a chosen sequence of dilation steps, the cumulated boundary lengths, as well as the number of clusters, may be calculated automatically and be stored.

Hence in a first step the borders of the buildings are extracted and their fractal behaviour is analysed. Then the pattern is dilated. For this aim each occupied point is surrounded by a black border, the size of which increases at each step of iteration. At the beginning (non dilated image), the reference element is the pixel. During the first dilation, each pixel is surrounded by a border of one pixel width. Then, the reference element is a square of $3^2$ pixels size. At the second iteration step, each pixel is surrounded by a border of two pixels width. The structuring element is then a square of $5^2$ pixels size. This procedure is then repeated for further steps. As the size of the surrounding squares gradually increases, the details smaller than the size of the structuring element are overlooked. At each step the boundaries of the dilated clusters are extracted.

For determining the fractal dimension of the boundaries there exist various standard methods. Several of these methods are implemented in *fractalyse* what allowed us to compare them and to test their pertinence. Recent investigations (De Keersmaecker *et. al.*, 2003; Frankhauser 2003; Frankhauser, 2004) made evident that one particular method, the correlation analysis, is

[7] This software has been developed by *Gilles Vuidel* in the frame of the contractual work for the *French Ministry of the Public Works*. If you want more information about *Fractalyse*, please consult the website of the research team ThéMA: http://thema.univ-fcomte.fr, heading *"Research teams"* -> *"City, mobility, territory"*.

particularly reliable for analysing urban patterns, compared to other ones. This incited us to recur to this method[8].

For applying this method, each black pixel is surrounded by a square of a given base length $\varepsilon$ and all black pixels lying within this square are counted. This allows to compute the mean number $N(\varepsilon)$ of black pixels lying within a vicinity of e of each black pixel. Then the base length of the squares is enlarged and again the same mean number is calculated. This procedure is repeated for a large range of values $\varepsilon$. Thus we obtain a series of values $N(\varepsilon)$ that can be represented on a Cartesian graph. The Y-axis corresponds to the number of counted elements $N(\varepsilon)$and the X-axis corresponds to the base length $\varepsilon$ of the squares, with $\varepsilon$ increasing from step to step. We call the graph $N(\varepsilon)$ empirical curve. For a fractal, the relation $N(\varepsilon)$ follows a power law. In order to establish a link between the empirical curve and the theoretical fractal relation, the next stage of the analysis is to fit the empirical curve with the theoretical one.

However we should expect that an urban pattern is not a pure fractal. Hence it seems useful to introduce a generalized fractal relation. Preliminary tests realized with *fractalyse* made evident that is reasonable to use the following law:

$$N(\varepsilon) = a\ \varepsilon^{D} + c$$

Let us give an interpretation of the three parameters:

1. $D$ is the fractal dimension.
2. $a$ is the so-called prefactor which globalizes possible deviations from the fractal law. Such deviations may have various reasons: the presence of big non-build-up areas may disturb the fractal law even if we tackle with pure fractal structures, but in a real-world pattern there may be observed local deviations from fractal law (Gouyet).
3. The parameter $c$ allows a correct adjustment of the curve if it seems useful to distinguish several ranges of $\varepsilon$-values for which we estimate the fractal dimension separately. Indeed it may be observed that in many cases there exist $\varepsilon$-ranges for which the fractal behaviour is very stable, but for certain critical distances $\varepsilon$ the shape of the empirical curve incites to suppose that the fractal behaviour changes. Then we may estimate the parameters for a low distance ranges and far distance ranges.

In any case a non linear regression is used to find the power law which best fits the empirical curve[9].

When considering the dimension $D^{(hSc)}$ we may expect that the more there are buildings of the same size and the more they are distributed in a homogeneous way, the fractal dimension tends to a value close to $D^{(hSc)} = 2$. When dilation reaches the limit of a unique cluster, the dimension usually drops down and tends to $D^{(hSc)} = 1$, since now neither fragmentation nor the

[8] Because the theory underlying the correlation analysis considers the simultaneous presence of two points at a certain distance, *i.e.* the mean distance between a pair of built-up pixels, the correlation dimension is a second order fractal dimension. In a multi-fractal theoretical framework, this correlation dimension should be extended to a series of three, four or more points.

[9] $D$ is often estimated by using a double logarithmic representation of the power law but here it has been chosen to minimise the least square deviations by means of a non-linear regression

tortuous aspect of clusters' boundaries subsist; the morphology of the envelope is rather smooth and is more or less a nearly linear, mono-scale object. The dimension $D^{(Sc)}$ the dimension will strictly measure the tortuous character of the boundary. Typical values will lie within the range $1 < D^{(Sc)} < 1.5$.

## 3.3 Two examples of application

We will now show how this concept could be applied to real world patterns. First we present the results obtained for an edge city of the metropolitan area of Stuttgart, Bietigheim-Bissingen. This town is situated in the Northern part of the agglomeration and offers a certain number of amenities. It is well connected to the regional capital of Stuttgart by a pertinent suburban railway and is close to a highway exit. Moreover the northern hinterland is sparsely urbanized and offers good opportunities for leisure activities. On the other hand important shopping centres are established nearby. Since about 30 years this town has considerably grown. As shows the name it was constituted at the origin of two towns which grew both in direction of the railways station situated between the two historical centres. This explains the horse-shoe like shape of the urban pattern.

On figure 13 we represented the extracted border from the original pattern as well as the boundary extracted after 15 dilation steps. Figure 14 shows the evolution of the length of the boundary across dilation.

**Figure 13:** ***The borders of the buildings of the town of Bietigheim-Bissingen, extracted from cartographic data.***

We observe that already after the first dilation step the length of the boundary decreases. This shows that fractal behaviour changes at the second step: for further steps the pattern looks more like a hybrid Sierpinski carpet than like a Fournier dust. The shape of the curve corresponds to a regular decrease of the boundary length as it is observed for hybrid Sierpinski carpets or even for Sierpinski carpets.

The number of clusters decreases also in a rather regular way. However we see that after 15 dilation steps there remain some isolated clusters in the hinterland of the town. Taking into account the shape of the pattern it doesn't seem useful to continue with dilating until one unique cluster appears: the typical horse-shoe like form of the urban pattern would then disappear. This shows that indeed a more realistic criterion for stopping dilation would be a relative stability of the number of clusters. In this example such a situation is observed approximately at the 12th iteration step.

The sequence of fractal dimensions which we have determined refers to the dimension $D^{(hSp)}$. We observe that the fractal dimension first increases, i.e. the smoothing due to dilation renders the cumulated boundary more homogeneous, small local clusters disappear progressively. Then the dimension decreases. This could us incite to think, that the cumulated boundary becomes more and more linear. However we must be aware that at the same time the number of clusters decreases dramatically (from 5707 at the beginning to 77 at the 10th dilation step) Thus dimension values of about 1.66 are still rather high for a topological linear object like boundaries.

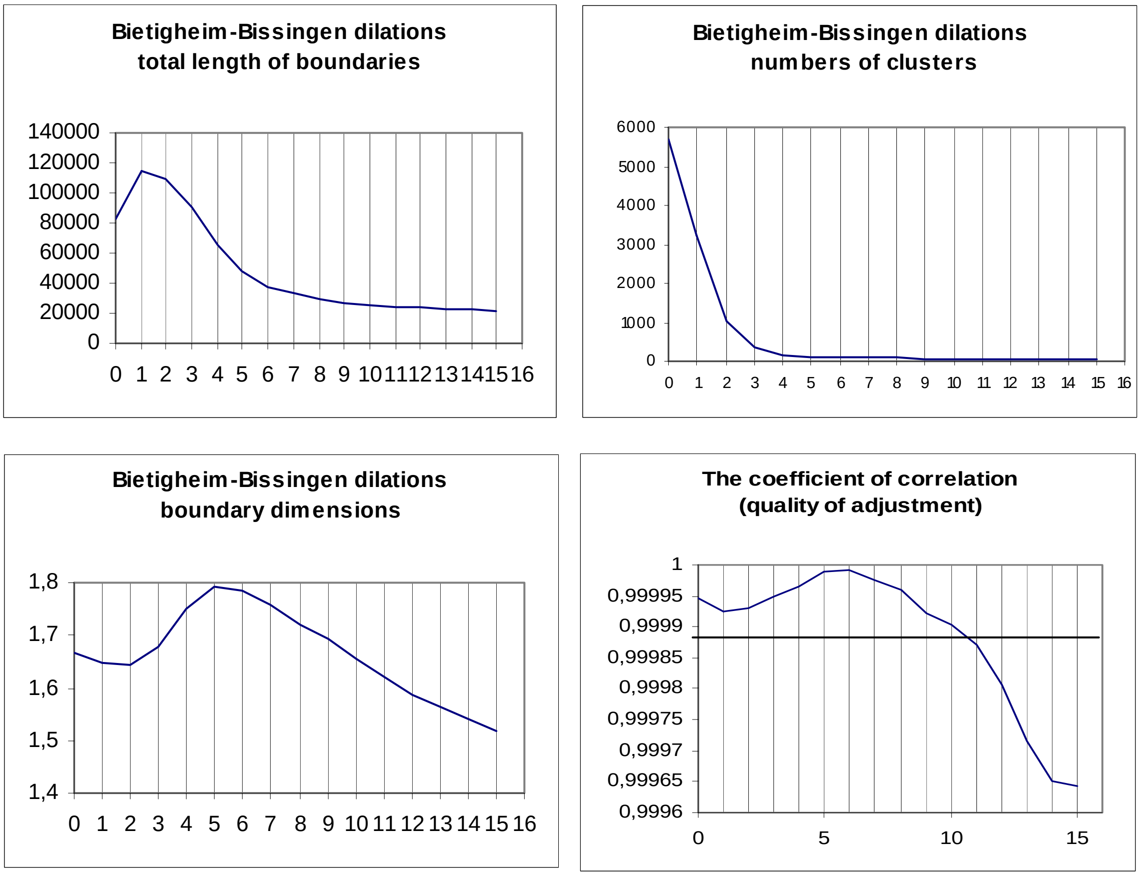



**Figure 14:** ***The results of the analyses obtained for Bietigheim-Bissingen (cf. text).***

The quality of adjustment for the estimation of the fractal dimension remains better than 0.9999 until the 10th dilation step. Experience shows that this is the lower limit for which we may assert that the adjustment is very pertinent. Thus we may conclude that for the range of dilations for which the adjustment is excellent, the fractal dimension $D^{(hSp)}$ varies finally in a rather small range. Hence the tortuous character remains rather stable in this range.

Similar results have been obtained for other kinds of urban patterns as e.g. shown in figure 15. Rixensart is an outskirt of Brussels and shows a quite different type of spatial organisation: the urbanization is rather diffuse as in many Belgian cases (cf. De Keersmaecker *et. al.* 2003).

However the shapes of the curves show similar characteristics: the dimensions increase first and decrease for higher dilation steps. Let us emphasize that in this example the quality of adjustment remains better that 0.99999 all over the dilations. The dimension values, which vary in for the range of first ten steps between 1.74 and 1.78, are in general higher that those of the first example, what is reasonable: the Rixensart pattern is more homogeneous then that of Bietigheim-Bissingen.

The number of clusters drops again down in a regular way and tends to reach constant values at about the 18th dilation step. The transition from the Fournier dust to the hybrid Sierpinski carpet occurs in this case at the 3rd/4th step. This may be due either to the particular data base (the size of a pixel may be different) or to the diffuse urbanisation.

***This fact that on the one hand $D^{(hSp)}$ seems to be restricted to rather small range for a given pattern and that on the other hand the values obtained for different patterns vary sufficiently shows that the dimension $D^{(hSp)}$ seems to be a rather good descriptor for the multi-scale link between border and envelope.***

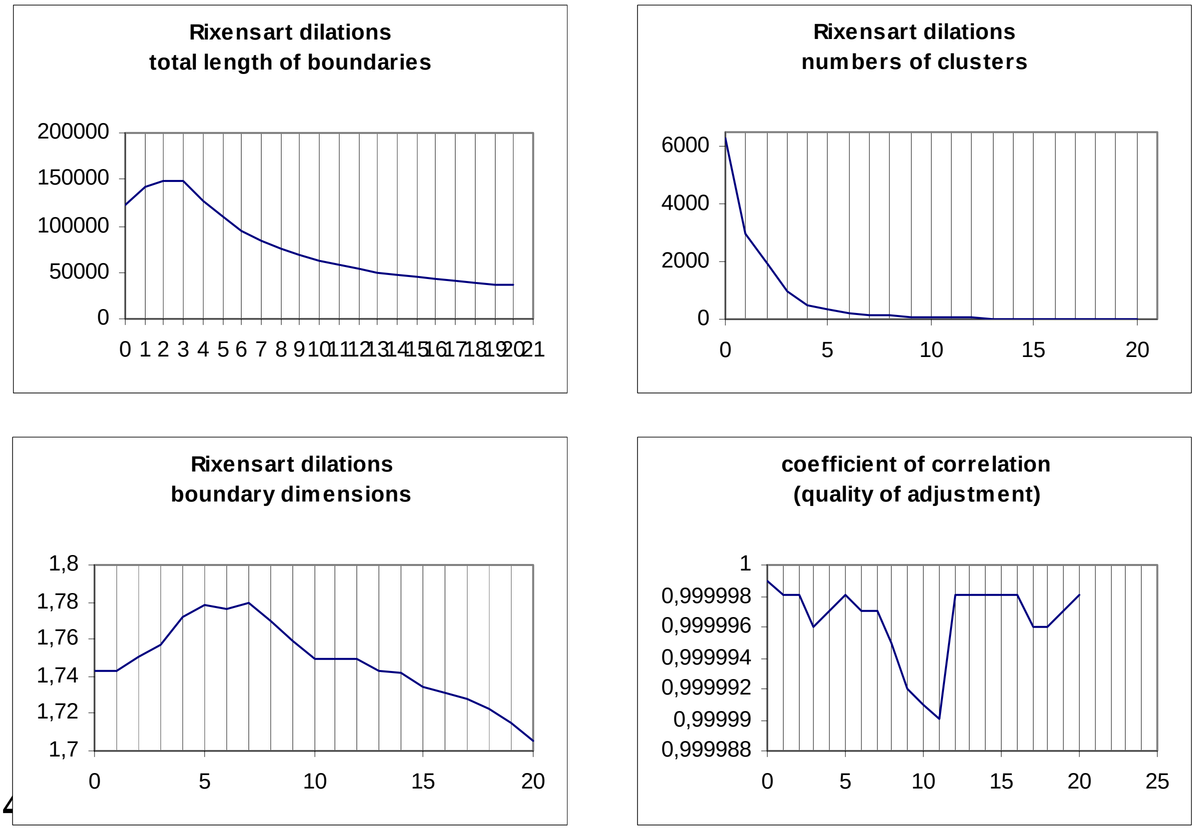



**Figure 15:** ***The results obtained for the analysis of Rixensart (cf. text)***

Delimiting urbanized areas is a difficult tool since urban sprawl generates more and more irregular settlement patterns. After having defined the notions of border and envelope, we have shown that by recurring to fractal geometry it is possible to develop a coherent concept of delimiting urbanized areas. Be recurring to the notion of covering a direct link may be established between the spatial distribution of buildings and the subsequent construction of the envelope. For this aim we start from the micro-scale of buildings and define a procedure which allows constructing stepwise larger and larger clusters of build-up areas for which the boundaries may be defined.

For real patterns this logic is replaced by a progressive dilation. The boundaries extracted at different steps allow observing to what extend the morphology of these virtual boundaries changes or not across scales. For each step the morphology of the boundary may be explored by means of fractal analysis. Thus beyond of the construction of the envelope deepen insight about the spatial organisation of urban pattern is obtained.

Further work should focus on clarifying if there exist for real world patterns morphological criteria the criteria for stopping dilation. Moreover it is intended to test to what extend the dilated patterns may be modified in order to conserve on the one hand the shape of the envelope generated, but to avoid on the other hand their “blown up” aspect which results of course directly from the procedure of dilating. This could help to improve the feature of the extracted envelope.

Until now only few dilation steps has been considered for a larger set of towns[10]. Of course it is intended to apply the method to a large set of settlement patterns and to study also the $D^{(Sp)}$-values for different dilation steps.

# 4. Conclusion

Delimiting urbanized areas is a difficult tool since urban sprawl generates more and more irregular settlement patterns. After having defined the notions of border and envelope, we have shown that by recurring to fractal geometry it is possible to develop a coherent concept of delimiting urbanized areas. Be recurring to the notion of covering a direct link may be established between the spatial distribution of buildings and the subsequent construction of the envelope. For this aim we start from the micro-scale of buildings and define a procedure which allows constructing stepwise larger and larger clusters of build-up areas for which the boundaries may be defined.

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[10] In Frankhauser (2003) the dimensions $D^{(hSp)}$ and $D^{(Sp)}$ of a certain number of towns are compared which refer however to boundaries extracted after few dilation steps.

Until now only few dilation steps has been considered for a larger set of towns[11]. Of course it is intended to apply the method to a large set of settlement patterns and to study also the $D^{(Sp)}$-values for different dilation steps.

## Acknowledgements and information about data bases

The authors are very indebted to the *French Ministry of the Public Works* for having financed in particular the development of the software package used for fractal pattern analysis *fractalyse*. We are also grateful to all institutions which provided the cartographic data used in the present context, in particular the municipality of *Besançon* (figure 1a), the *Communauté Urbaine de Lille* (figures 1b and 4), the municipality of *Stuttgart* (figure 2a), the *Agence d'Urbanisme et de Développement de la Communauté d'Agglomération du Pays de Montbéliard* (figure 9) as well as to the *Städtebauliches Institut* of the University of Stuttgart which has conceived the data base used for figure 2b. The data sources used for figure 13 have been created by the *Landesvermessungsamt Stuttgart* and that of the outskirts of Brussels are digitized cartographic data provided by the *Belgium National Geographic Institute (I.G.N.)*.

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