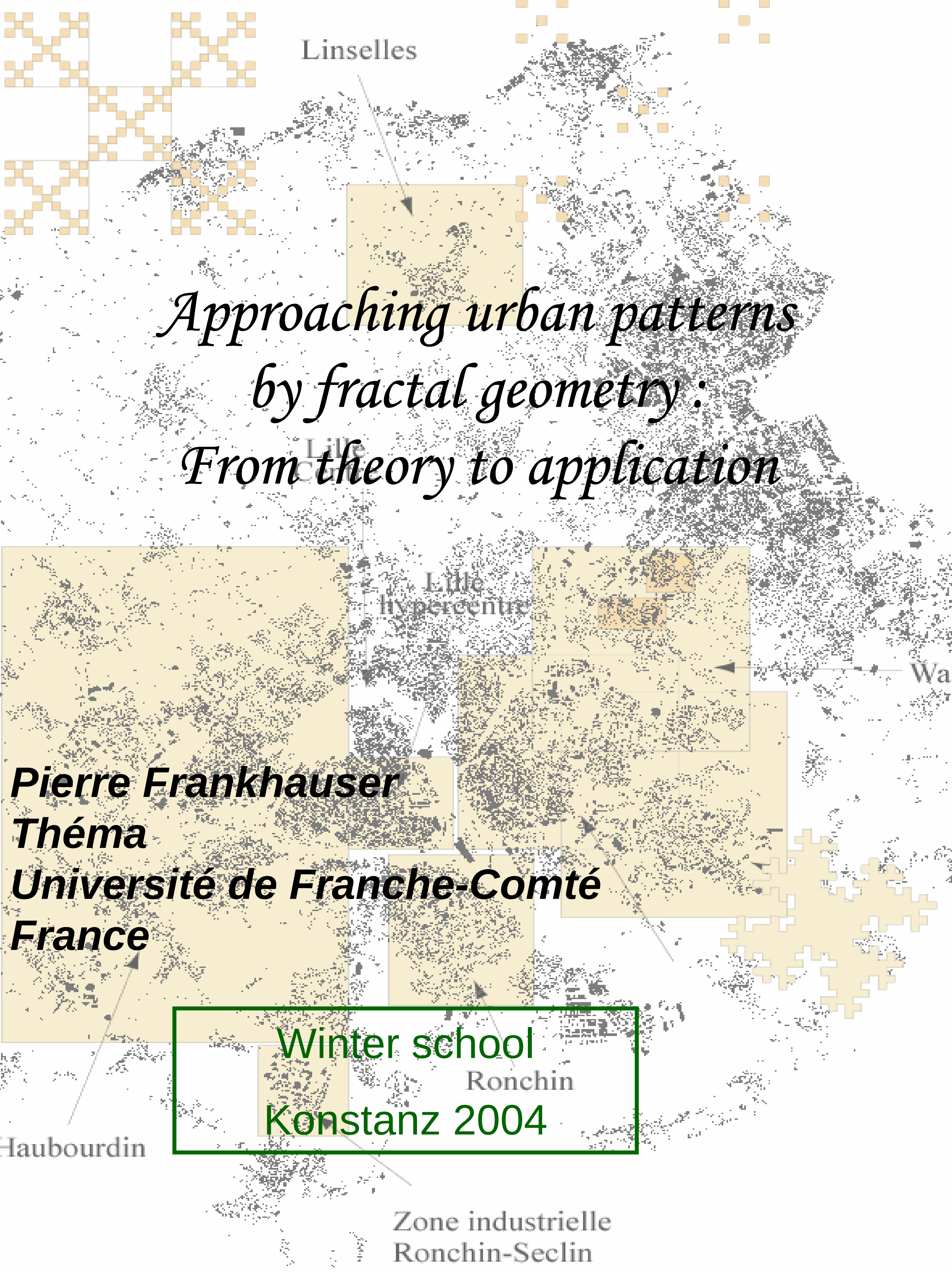


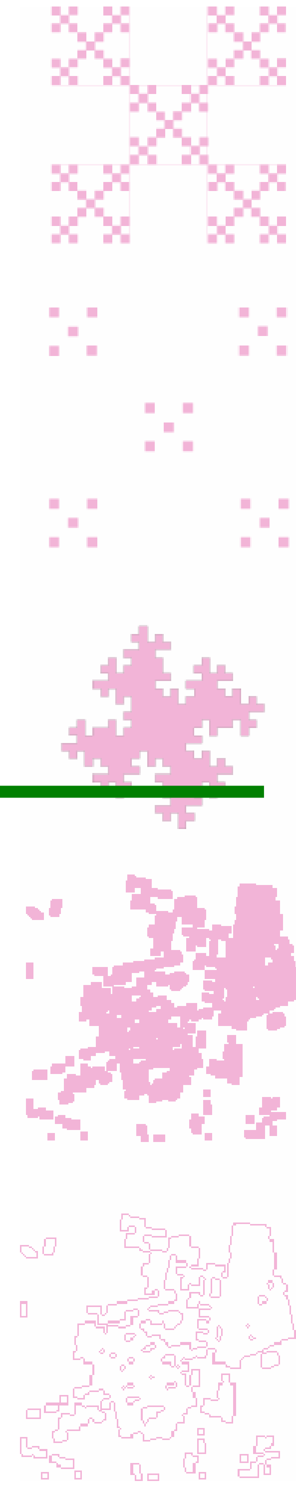
Approaching urban patterns by fractal geometry : From theory to application

Pierre Frankhauser
Théma
Université de Franche-Comté
France

Winter school
Konstanz 2004

Fractal geometry

An introduction



fractal geometry and fractal analysis

the fractal approach is first a geometric approach and not measuring method

↓

the fractal approach is mainly an approach for spatial modelling allowing to introduce methods for analyzing forms

↓

close interaction between modelling and mesuring

↓

measuring methods imitate modelling

↓

comparison between spatial models and empirical structures

- ***the main features of fractal structures***
- ***the measuring methods***
- ***application to urban patterns***
  - ***morphological analysis***

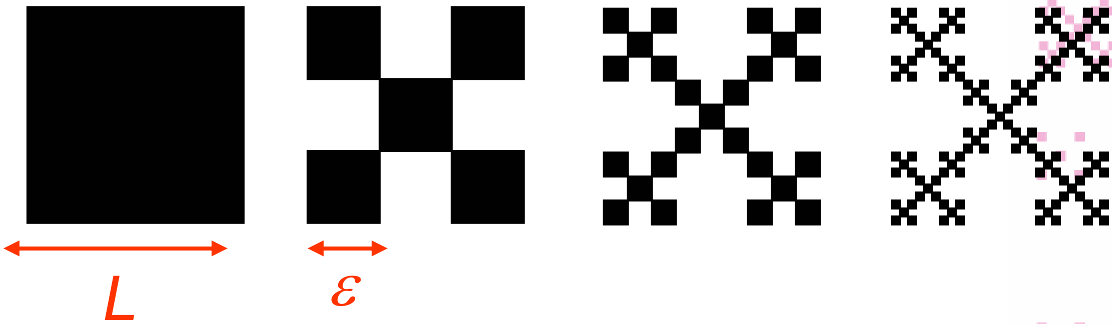
    what means fractality for urban patterns
  - ***conceptual reflections***

    From theoretical reflexions to planning purposes

how to construct a fractal

generation by iterative mapping

example : a Sierpinski carpet



initiator generator

$N = 5$, $\varepsilon = 1/3\ L = r\ L$

1st step	2nd step	3rd step
$N_1 = N$	$N_2 = N^2$	$N_3 = N^3$
$\varepsilon_1 = r\ L$	$\varepsilon_1 = r^2 L$	$\varepsilon_1 = r^3 L$

two linked geometric series

$N_n = N^n$ $\varepsilon_n = r^n L$

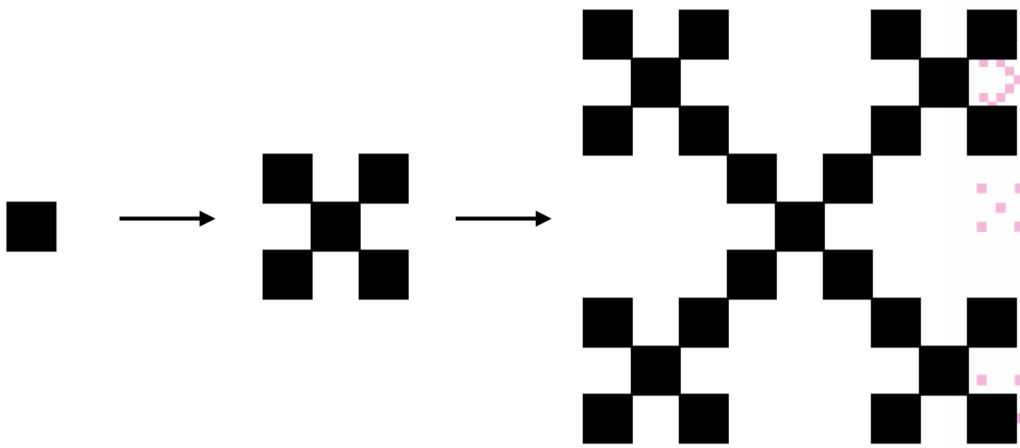
eliminate *n* → **fractal law**

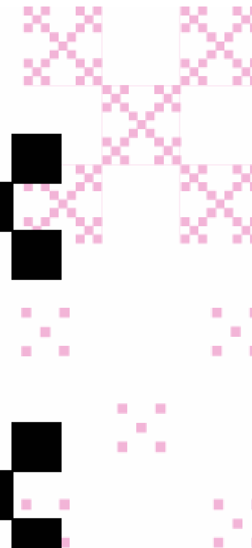
$$N_n = \varepsilon_n^{\ D}$$

fractal dimension

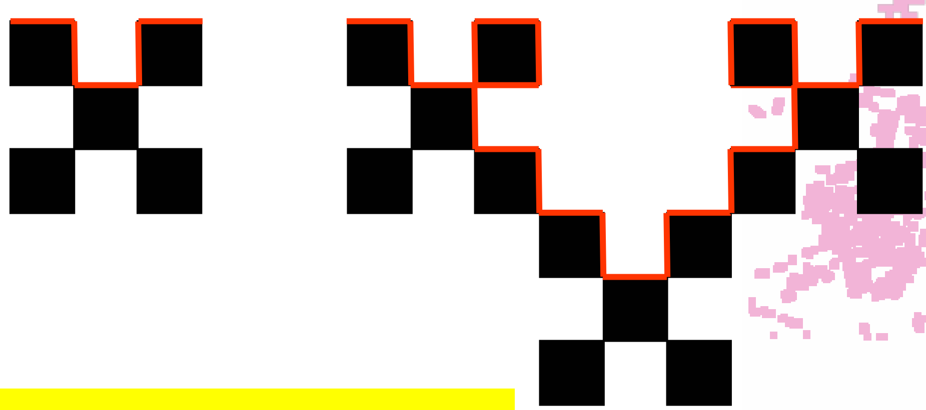
$$D = -\frac{\log N}{\log r}$$

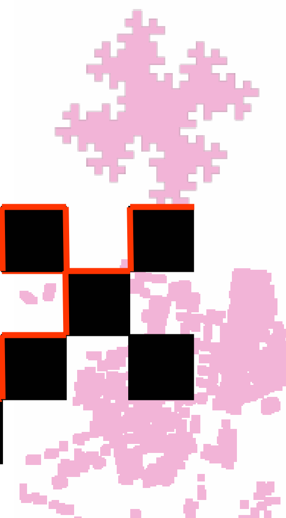
an alternative way of construction





fractal border

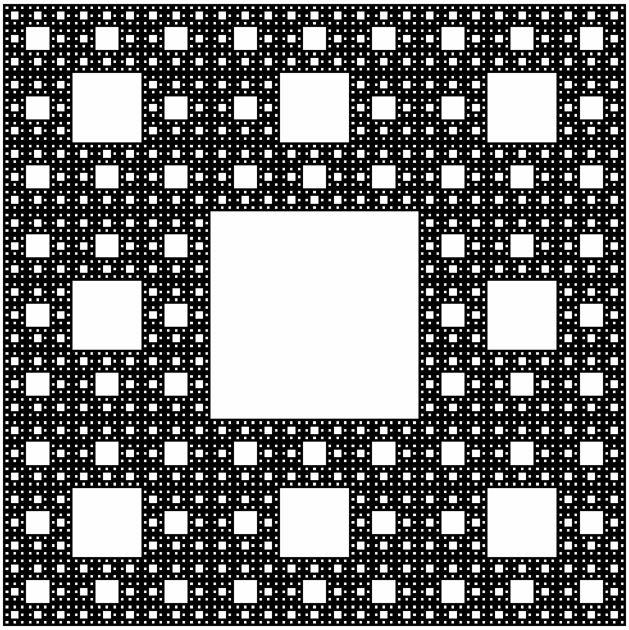


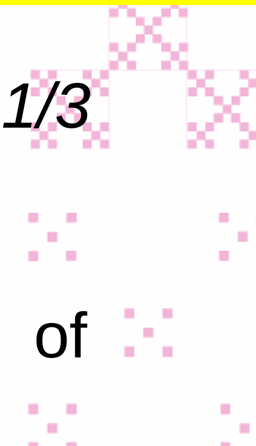


the same parameters *N, r*

border and surface have the same dimension *D*!

every fractal shows a spatial hierarchy, at least for the system of lacunes …

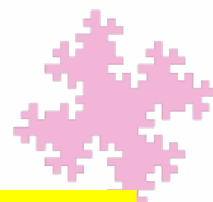




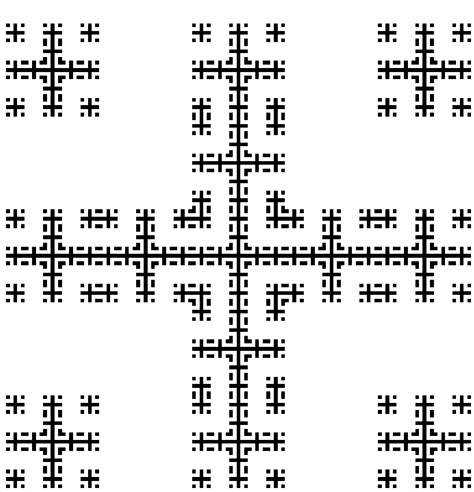
N = 8, *r* =1/3

D = 1,89

hierarchy of lacunes



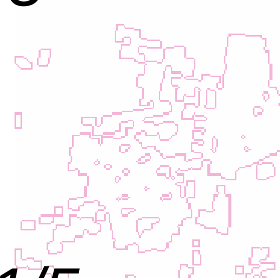
… but a hierarchy of clusters may be generated, too



central cluster:

N = 9, *r* = 1/5

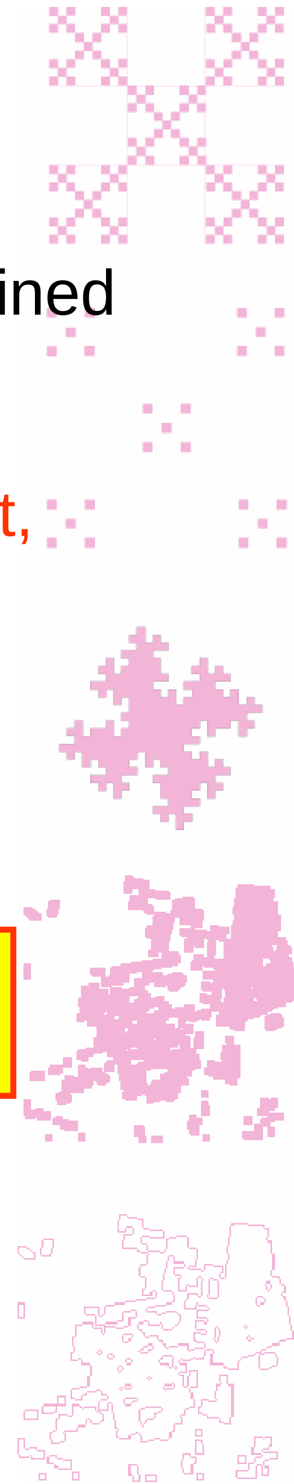
D = 1,37



total fractal:

N = 13, *r* = 1/5

D = 1,59



- the spatial hierarchies are generated by the iteration
- they follow strictly a well-defined hyperbolic distribution law:

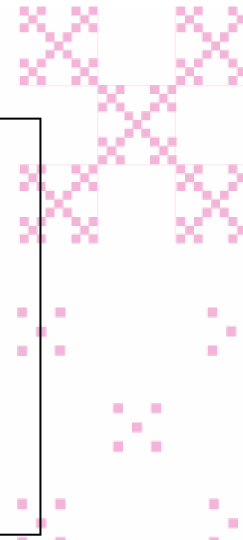
$$N(\varepsilon) \sim \varepsilon^{-\alpha}$$

number of lacunes — size

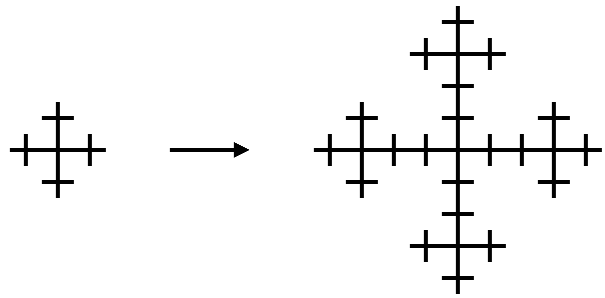
scaling exponent, eventually different from *D*

Pareto-Zipf distribution

fractals of linear topography

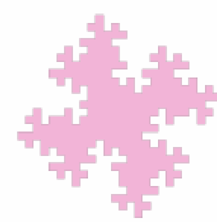


ramified fractals

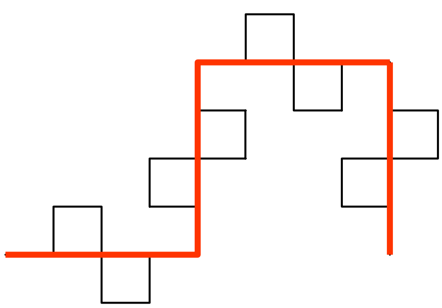


D = 1,46 (= Sierpinski carpet)

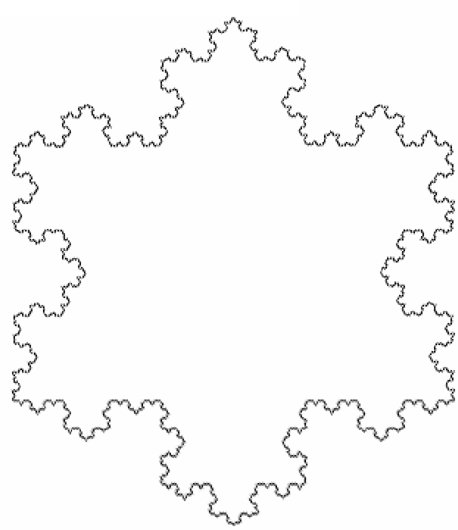
hierarchy of branches

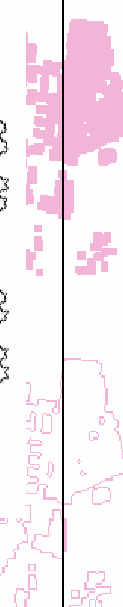


Dendritic, tortuous fractals



part of a teragon
D = 1,5



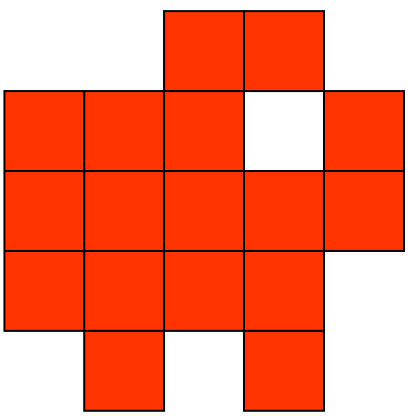


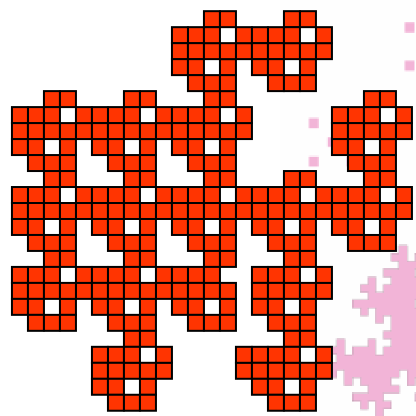
curve of von Koch
D = 1,26

hierarchy of bends

link with real world situation: the random fractals

A less symmetric generator …

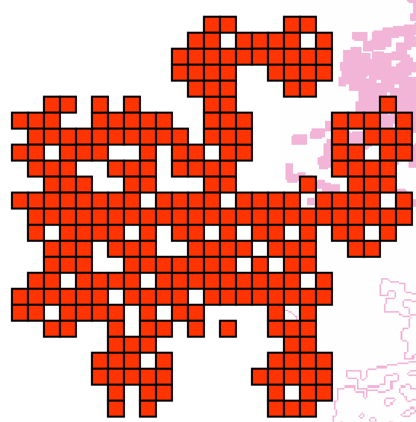




strict iteration

… and a more arbitrary position for the following steps

the already generated lacunes must be respected !!



random variation for 2nd step

a fractal is neither dense nor diluted, it is more or less contrasted

both uniformity and concentration are limit cases

" irregular" structures correspond eventually to a particular – fractal – order principle

such structures may be quantified

***Nicolis:* "a very simple model for complexity"**

the information given by the fractal dimension:

surface : degree of mass concentration across the scales (e. g. build-up mass)

spatial hierarchy

***D* = 0 concentration in a solely point**

***D* = 2 uniform mass distribution**

***D* low highly hierarchised (contrasted)**

***D* high low hierarchy**

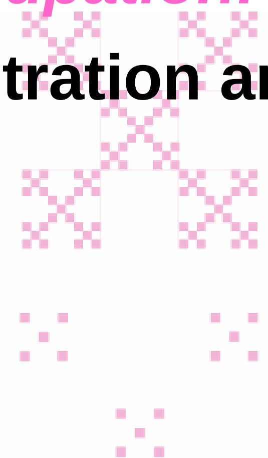
network: degree of accessibility / ramifications

***D* low very hierarchised, unequal accessibility (contrasted)**

***D* high low hierarchy → uniform**

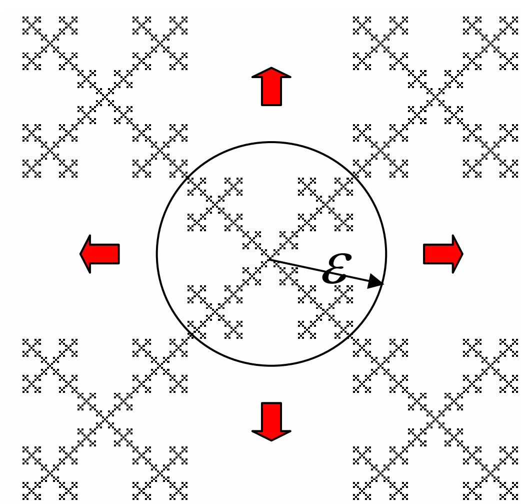
border : D > 1 tortuous/dendritic

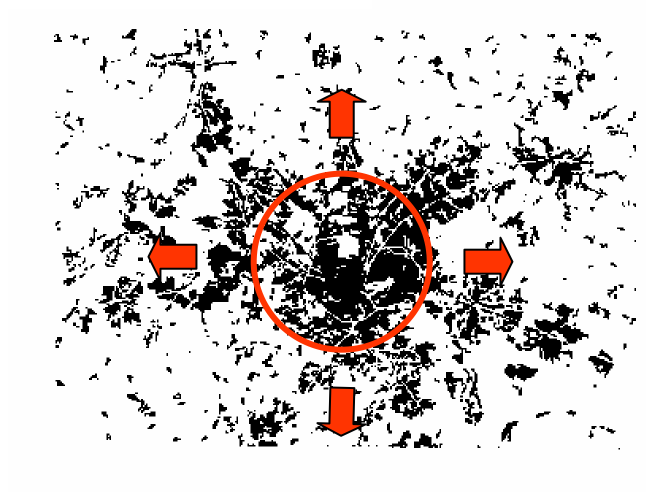
Information about the surface occupation:



measures the decrease of concentration and the contrast in a pattern

build-up surface : **mesures the radial dilution of builp-up mass in the vicinity of a building**





relative change of density :

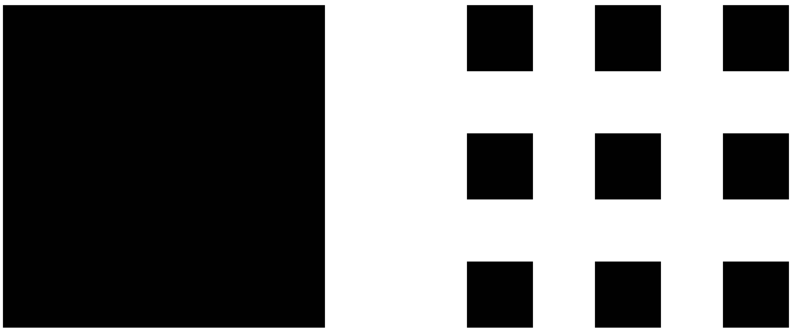
$$\frac{d\rho}{\rho} = (D-2)\frac{d\varepsilon}{\varepsilon}$$

variation relative de la densité

fractal dimension

variation relative de la distance

uniform distribution D = 2



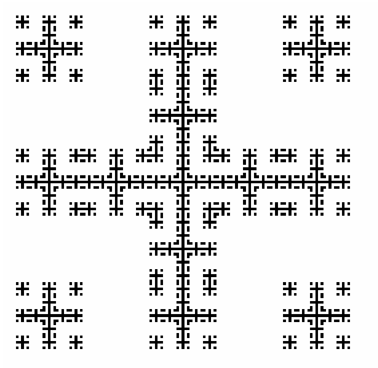
repetitive
and regular

lacunes of
same size

Brussels
Individual
housing estate
D_{cor} = 1.95

Brussels
pericentre
dense
D_{cor} = 1.97

Contrasted patterns: lower dimension values



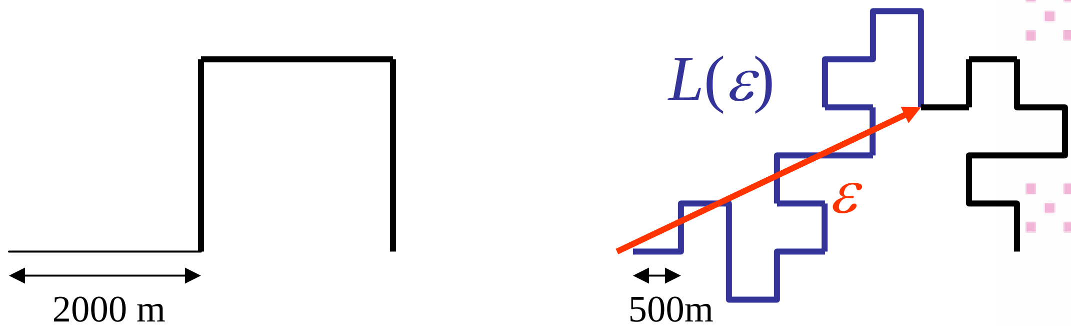
Contrasted fractal
D=1.59

Montbéliard
Valentigney
D=1.75

Cergy-
Pontoise
D=1.73

Information for border length

dendricity of borders across scales



relative lengthening of border length :

$$\frac{dL}{L} = (D+1)\frac{d\varepsilon}{\varepsilon}$$

relative variation of the length — ***fractal dimension*** — **relative variation of the distance**

The measuring methods

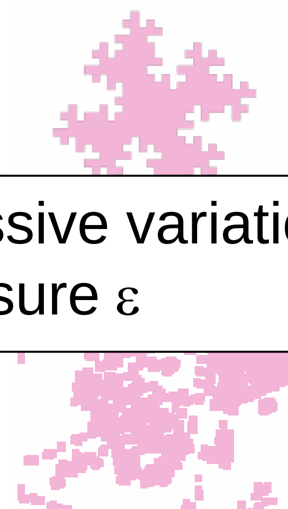
the general procedure for measuring fractal behaviour

the empirical structure is interpreted as "random fractal"

⇨ *no knowledge about a construction rule*

general method

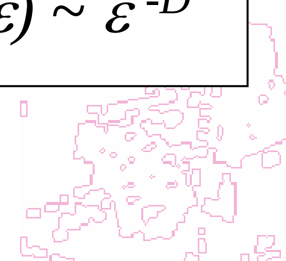
basic principle:



Iteration step *n* measure ε_n	→ *substituted by*	progressive variation of measure ε

fractal relation :

$N_n \sim \varepsilon_n^{-D}$	→ *substituted by*	$N(\varepsilon) \sim \varepsilon^{-D}$



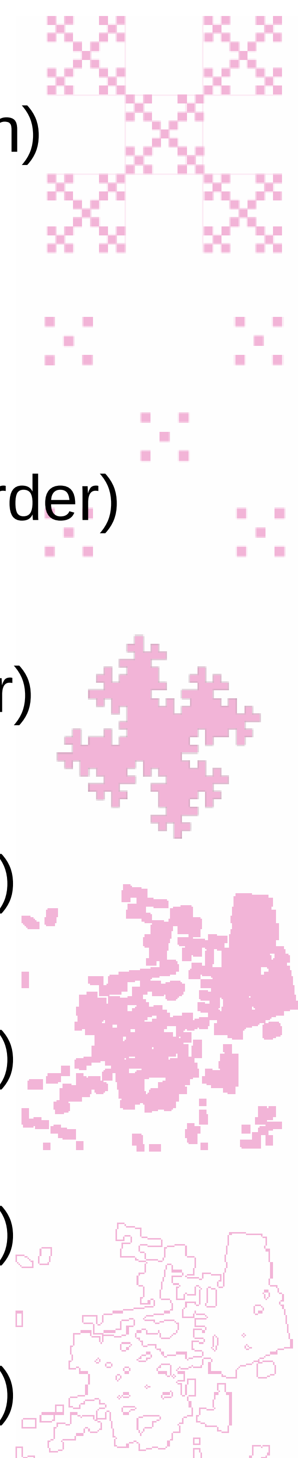
bi-logarithmic form

$$\log N(\varepsilon) = const - D \log \varepsilon$$

linear relation

The measuring methods

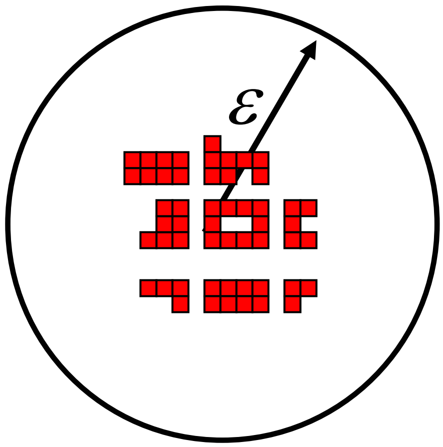
the measuring methods:



- radial analysis (local approach)
- analysis of Richardson
  (local approach)
- correlation analysis
  (global approach of second order)
- dilation analysis
  (global approach of first order)
- grid analysis
  (global approach of first order)
- box counting
  (global approach of first order)
- Gauss convolution analysis
  (global approach of first order)
- variogramme
  (global approach of first order)

… and each other multi-scale method

radial analysis



counting the number $N(\varepsilon)$ of occupied sites localized at a distance ε from a chosen counting centre

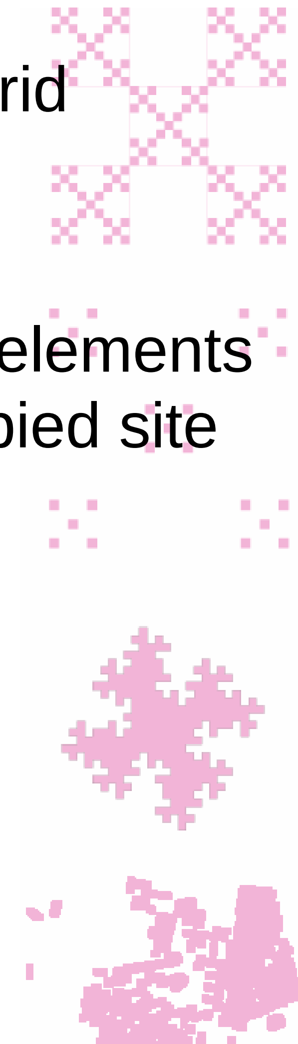
fractal law:

$N(\varepsilon) \sim \varepsilon^D$

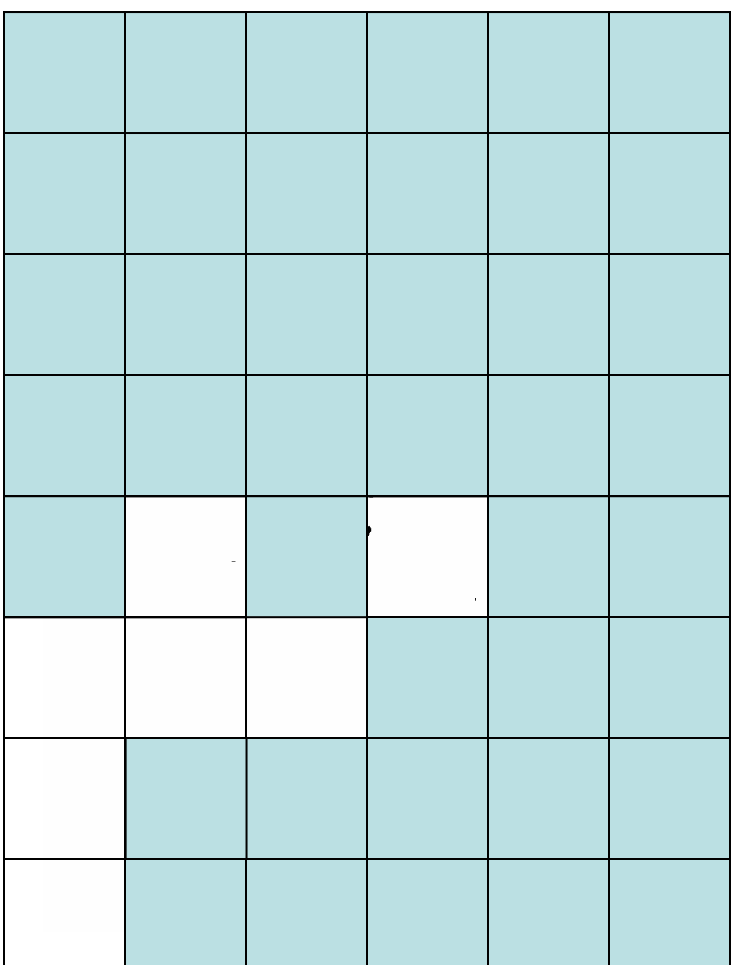
local information

no data transformation

grid analysis



- covering the structure by a grid
- varying the size ε of the grid elements
- counting the number $N(\varepsilon)$ of elements containing at least one occupied site
- fractal relation : $N(\varepsilon) \sim \varepsilon^{-D}$



global information

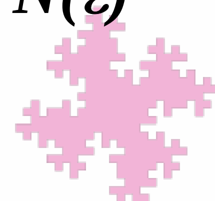
no data transformation

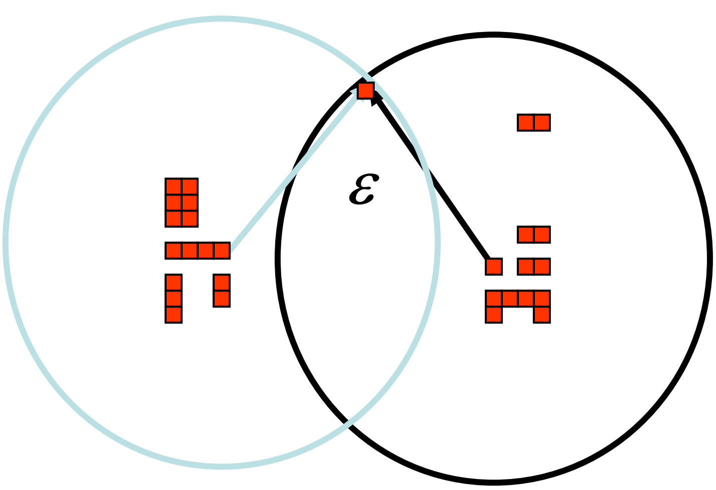
correlation analysis

the same logic as the radial analysis, however :

☞ **the counting procedure is realised at each occupied site**

☞ **computing the mean number $N(\varepsilon)$ of occupied sites for each distance ε (number of "correlations")**





D_{cor} : dimension de second ordre

global information

no data transformation

Dilation analysis

stepwise dilation of texture

- ⇨ **smoothing procedure**
- ⇨ **stepwise loss of details**

texture

2e step

$N(\varepsilon)$: **Number of squares of size** ε **nessary to cover the black surface**

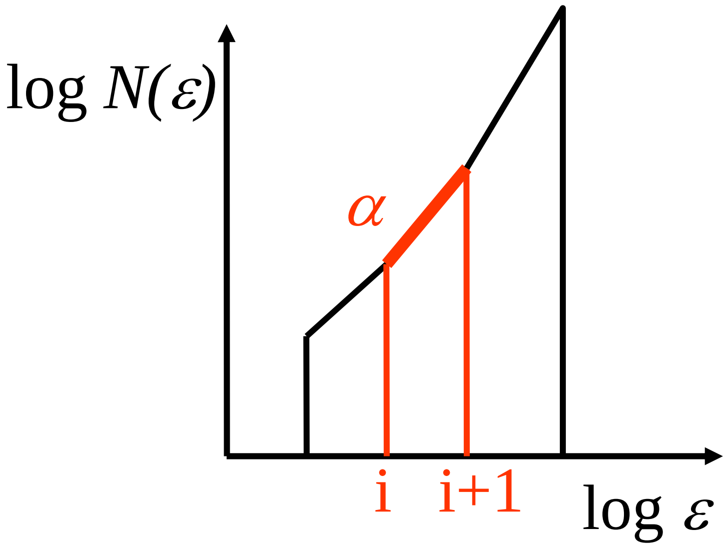
global information

data transformation

The measuring methods

type of analysis	***local global***	***border L surface A***	***reliability and application***
radial	L	L and A	good *specific logic*
grid	G	L et A	not reliable
box counting	G	L et A	very reliable, realisation difficult
correla-tion	G	L et A	very reliable
dilation	G	L et A	L : not reliable A : with caution
gaussian convo-lution	G	L	L : good A : difficult

Curve of scaling behaviour

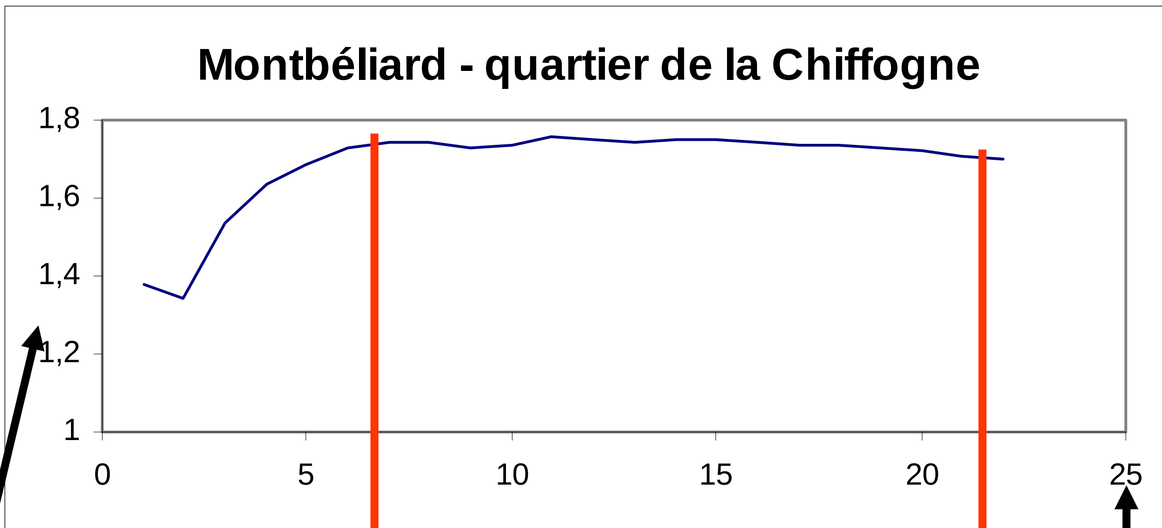


slope in the bi-log représentation

$$\alpha = \frac{\log N(i+1) - \log i}{\log(i+i) - \log i}$$

good indicator for identifying the ruptures in the fractal bahaviour

Example of a curve of scaling behaviour for a correlation analysis

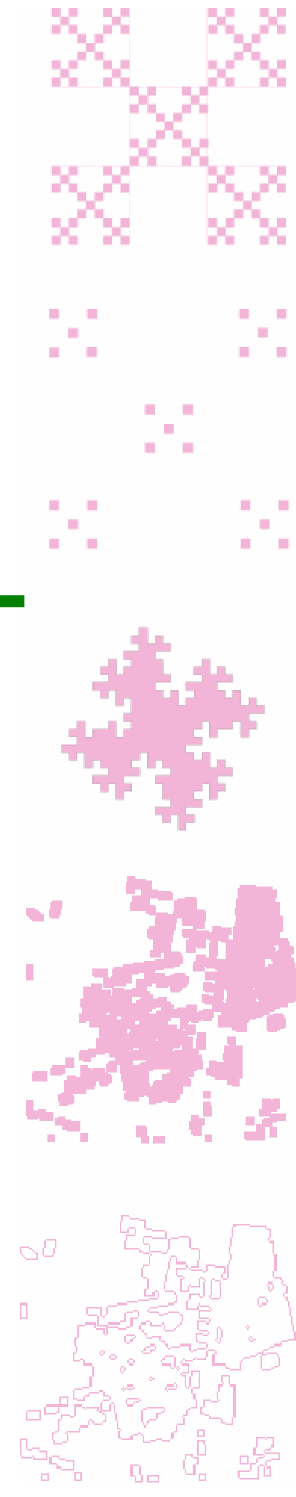


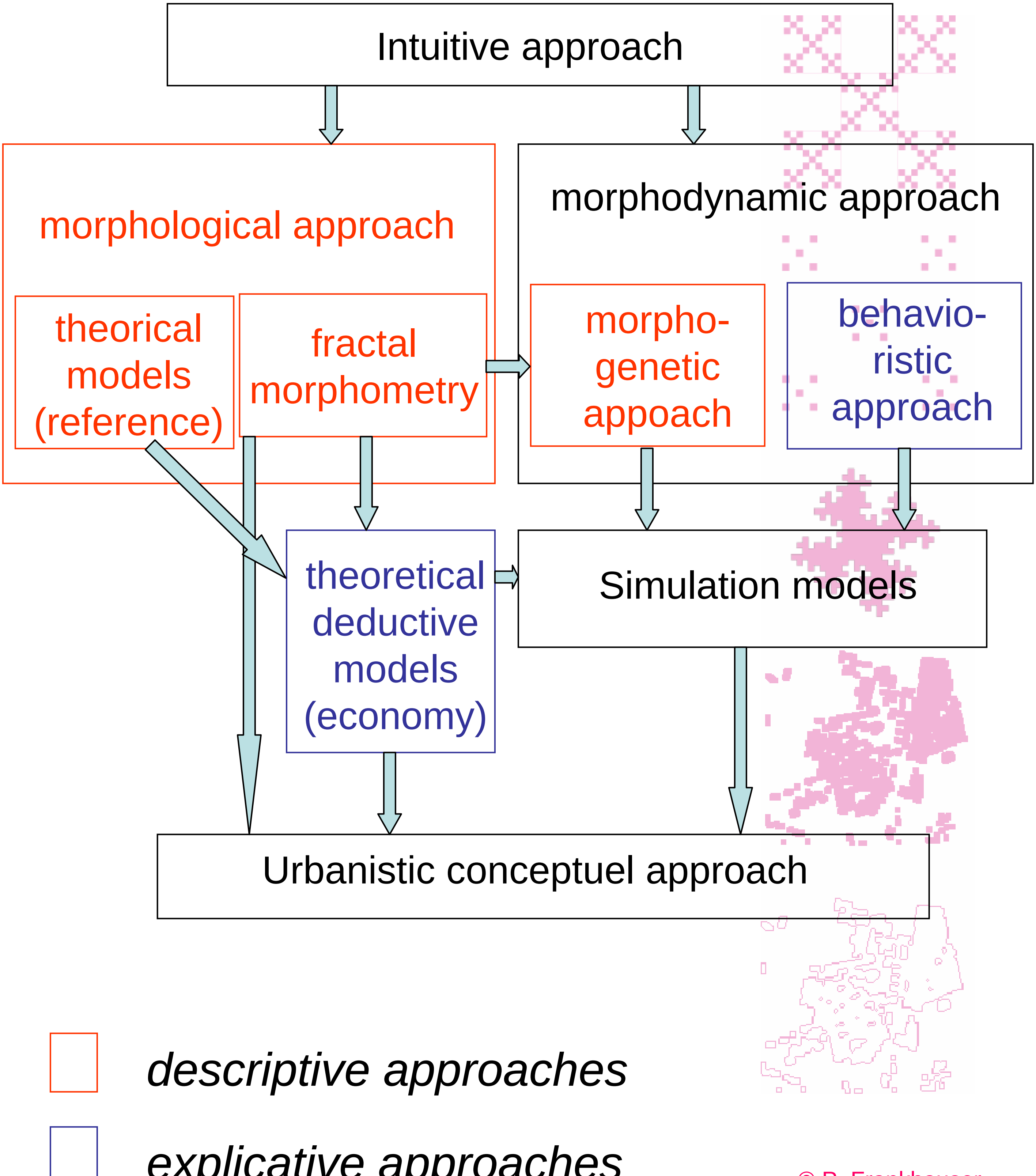
stable range for the estimation

scaling exponent α

distance parameter ε

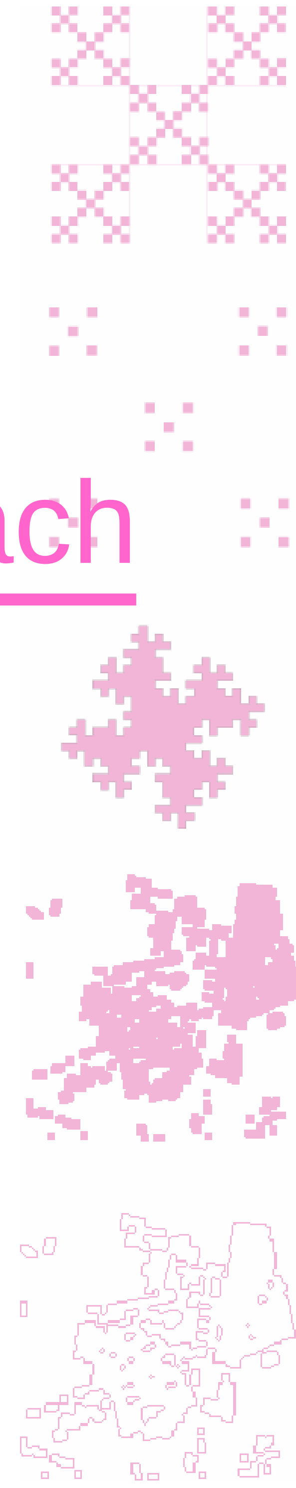
Urban patterns and fractality



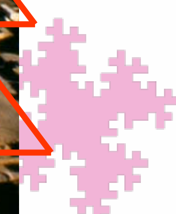

Intuitive approach
morphological approach
morphodynamic approach
theorical models (reference)
fractal morphometry
morpho-genetic appoach
behavio-ristic approach
theoretical deductive models (economy)
Simulation models
Urbanistic conceptuel approach
descriptive approaches
explicative approaches

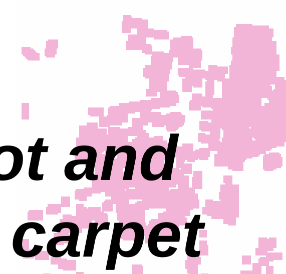
- intuitive approach
- theoretical reference models
- morphological analysis
- morphogenetic approach
- theoritical deductive models
- simulation models
- conceptual approach for urban planning

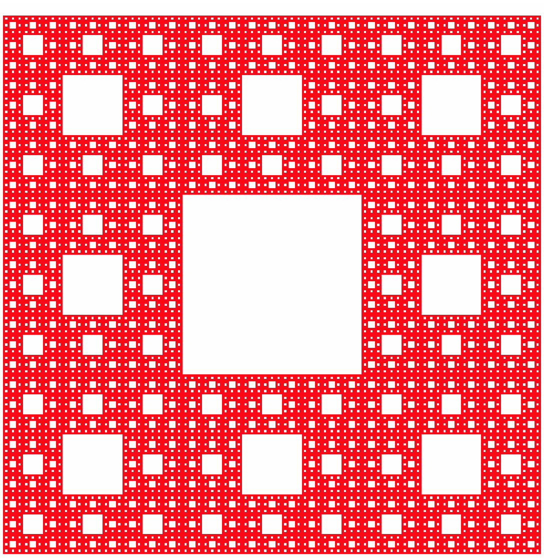
An intuitive Approach



hierarchies in urban space

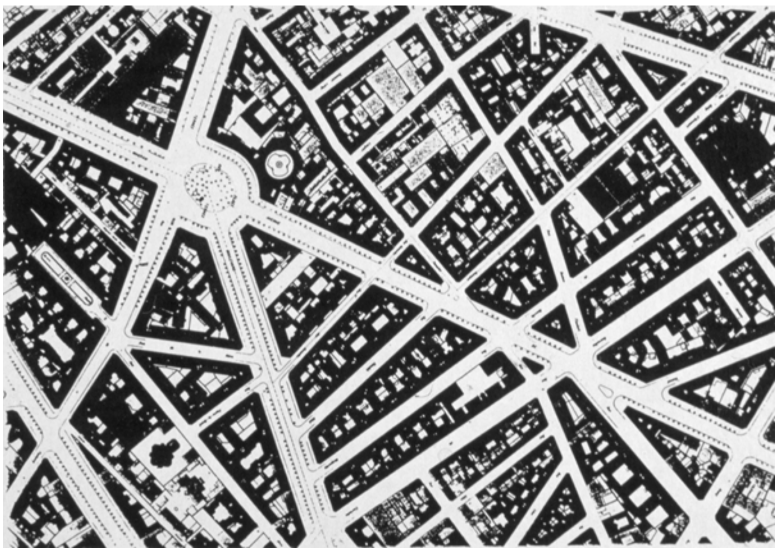


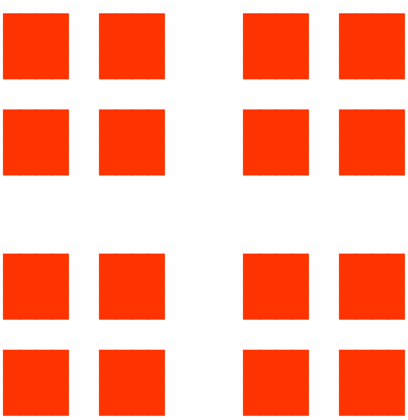


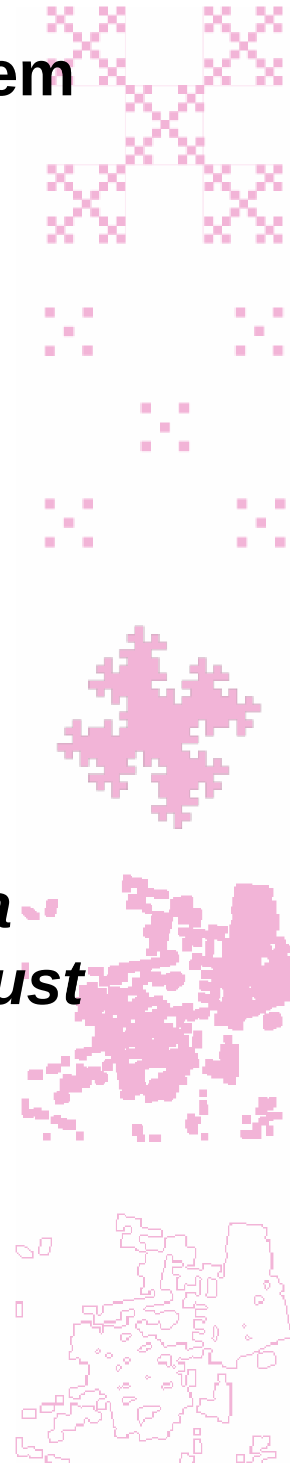


Nouak Chot and Sierpinski carpet

hierarchies of the street system

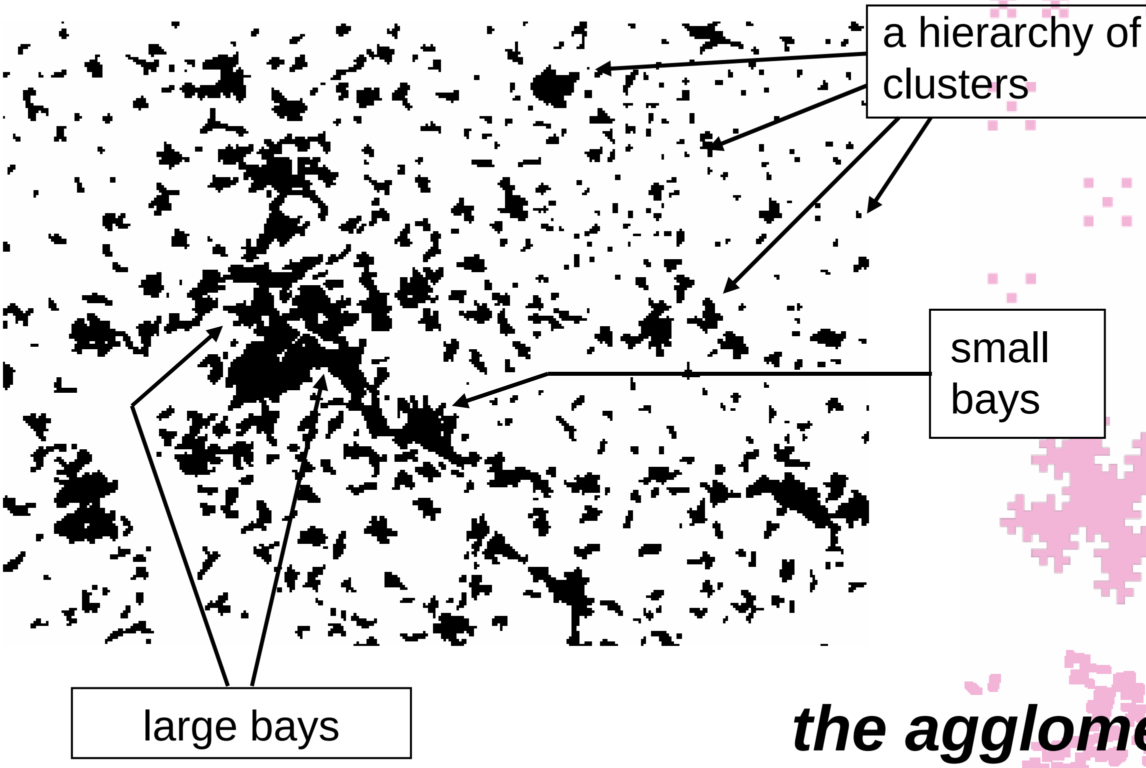


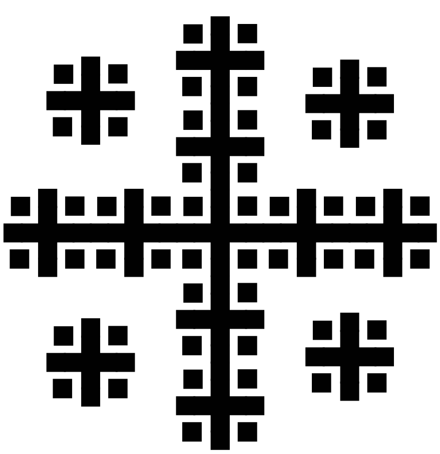


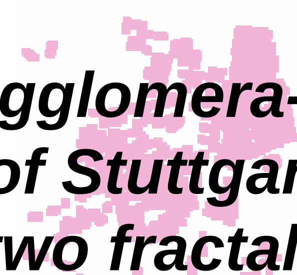


Paris and a Fournier dust

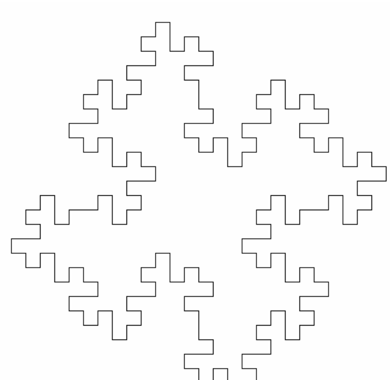
multi-scale organization of urban patterns

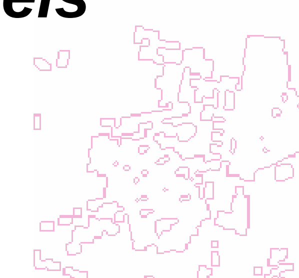






the agglomeration of Stuttgart and two fractal models





Different fractal models serving as references …

… according to the domain of application

⇨ **these models play the same role as cercle, squares etc. do, when referring to traditional geometry**

the domain of application conditions

⇨ **the choice of appropriate methods**

⇨ **the inetrpretation of the results**

for urban patterns :

⇨ **3 models to analyse the spatial distribution of the build-up surface *(urban pattern analysis)***

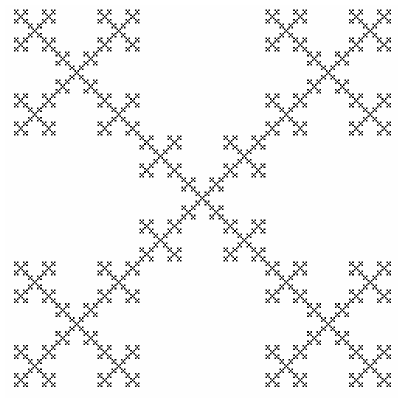
⇨ **one model to make evident the dendricity of urban borders**

the models

different types of Sierpinski carpets

one unique cluster

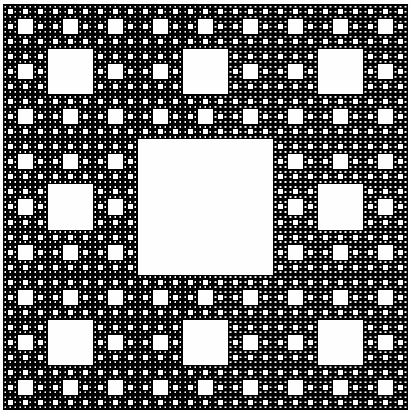
$D_s = D_{b\text{-}tot} = D_{b\text{-}clust} = 1.46$



one unique cluster

$D_s = D_{b\text{-}tot} = 1.89$

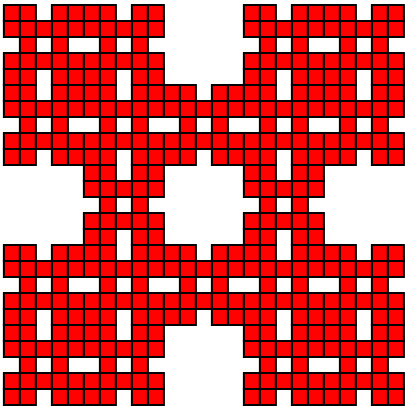
$D_{b\text{-}clust} = 1 < D_{b\text{-}tot}$



one unique cluster

$D_s = D_{b\text{-}tot} = 1.72$

$D_{b\text{-}clust} = 1.21 < D_{b\text{-}tot}$

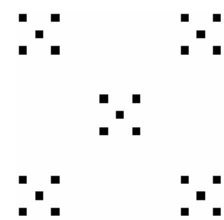


the Fournier dusts

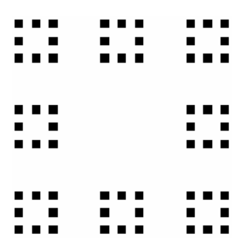
A series of non-uniformly distributed clusters

More or less uniform

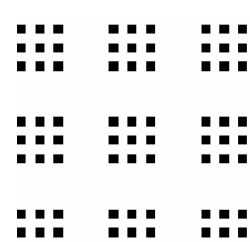
$D_s = D_b = 1$



$D_s = D_b = 1{,}29$



$D_s = D_b = 1{,}36$



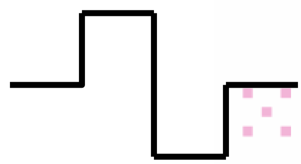
le téragone

$D_s = D_b$

bordure fractale

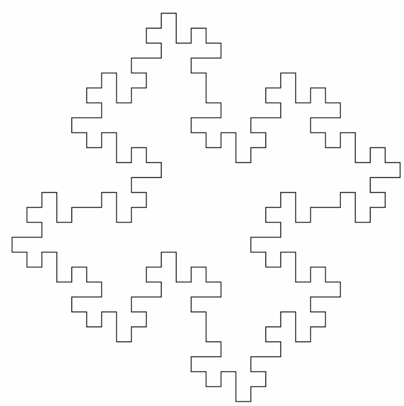
compact à l'intérieur

le générateur



bordure du téragone

$D_b = 1,5$



et la surface

$D_s = 2$